If there comes a little thaw,
Still the air is chill and raw,
Here and there a patch of snow,
Dirtier than the ground below,
Dribbles down a marshy flood;
Ankle-deep you stick in mud
In the meadows while you sing,
"This is Spring."

—C. P. CRANCH. "A Spring Growl."

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SCHOOL SCIENCE
AND
MATHEMATICS

NEWER ASPECTS OF THE POLIOMYELITIS
PROBLEM*

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With the publicity which poliomyelitis has had in recent years it seems hardly necessary to define or describe the disease which we are to consider today. Poliomyelitis is a common virus disease which usually runs a mild course characterized by upper respiratory or gastrointestinal symptoms but which may be complicated by central nervous system involvement and demonstrated by paralysis even to the extent of causing death.

The symptoms of poliomyelitis may vary and most of the cases are so mild that the diagnosis is not established at the time of infection but only can be proven to have occurred by blood tests. A second classification of polio involves obviously sick individuals with nervous system involvement, but which are non-paralytic throughout their disease. The most well-known form of polio is the paralytic disease in which the legs, arms, diaphragm and other muscles are effected. The damage done by the virus acting upon the motor nerve cells or anterior horn cells of the spinal cord or mid-brain is the direct cause of the loss of use of the voluntary muscles.

Poliomyelitis is used as the name of this disease now rather than the term "Infantile Paralysis." This is due to the fact that in recent years the highest disease incidence has changed from infants to older children and many adults are victims. In fact adults account for the most serious cases. This surprisingly is a result of improved sanitary

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conditions. In earlier times children and infants were infected early in life and most of the cases were "inapparent" whereas now the infants are not exposed in our highly sanitized world and the older children or adults acquire the more severe form of the disease. Paralytic involvement is now often seen in these age groups.

The significance of this disease is not nearly as great as several other diseases but since many people are incapacitated as a result of polio infection the public has become highly aware of the existence of this infection. The activities of the late president Franklin D. Roosevelt, the "March of Dimes" which he started, and the National Foundation for Infantile Paralysis have further accentuated this public knowledge of polio. Non-paralytic cases far outnumber the paralytic cases and most everybody has one or more polio infections during his lifetime.

Historical records would indicate that infantile paralysis or polio has existed for a long time yet it took from 1840 when Heine first described the syndrome until 1908 before any progress was made. In 1908 the disease was first demonstrated in monkeys inoculated with human material. In 1909 the transmission of the disease from animal to animal was proved. Within a year it was soon learned that infection caused a resistance to the disease and that infected animals produced a substance in the blood which would neutralize or inactivate the agent. The agent was found to be a virus or disease agent which was submicroscopic and would pass through a filter which would retain bacteria. Progress was very slow from 1912 until 1937 mainly due to the lack of experimental animals. Monkeys and humans were thought to be the only susceptible animals. In 1939 Armstrong adapted the Lansing strain of polio virus to mice and this resulted in renewed research activity.

During the period from 1940 until 1949 several important advances were made. Three distinct types of viruses were found to be capable of causing the disease. Confirmation of the early work of Trask who found the virus in sewage led to the proof of the existence of virus in high titer in the feces of infected humans and animals. The discovery of fecal virus suggested an alimentary pathway of infection and a different mode of spread in the body. Chimpanzees fed virus were found to excrete virus in the feces.

In 1949 the avalanche of new information began to be published. The recent advances are primarily the result of new tools which became available. Most notable of these was the introduction of tissue culture methods for virus study. The cultivation of cells or tissues in flasks, bottles, and test tubes was made possible on a much larger scale by the use of antibiotics which were capable of keeping contamination to a minimum.

Funds provided by the collections of the National Foundation for Infantile Paralysis were distributed to many research institutions and this resulted in an increase in the number of scientists working on polio from 50 to 75 in 1940 to many times that number in 1954.

Demonstration of the existence of the virus in the blood stream by several virologists raised hopes that there was a possibility of serum or vaccine prophylaxis of the disease. Other investigators have turned to genetic studies with the hope of obtaining the appropriate strains of polio viruses for vaccine production. Several groups have worked with viruses hoping to find a strain or strains which even though alive produce protection but no disease. Although the main advances have been in the laboratory and in the search for a vaccine some progress has been made in the therapy of the victims of the disease. Each of these phases of polio warrant consideration but with the limit of time available to me I must only attempt to emphasize the most important.

Tissue culture methods have by far done the most for polio research. Dr. John F. Enders, this year's Nobel prize winner in medicine was the first person to do virus studies in tissue cultures in a practical way. He utilized human tissues for the cultivation of polio virus. Soon after this initial report monkey tissues were tried and found to permit a rapid and significant growth of virus. Kidney tissue yields the highest amount of virus. The HeLa strain of epithelial cells derived from a human carcinoma has been successfully used for the cultivation of the polio viruses. The nutrition of tissues grown in these "in vitro" conditions is very important and at present a number of synthetic solutions are capable of supporting tissue growth. Most notable of these is that of Morgan, Morton, and Parker and called M199. This mixture contains approximately sixty amino acids, vitamins, salts, and growth factors. Bathing or feeding tissues with such media permits the survival and some growth of cells. One then can infect certain of these cells with polio virus and obtain reproduction of a great magnitude. One virus particle can be made to yield thousands of particles.

Tissue culture methods have been highly advantageous in studying the epidemiology of this disease. Virus isolations can be made from blood, feces, etc. of patients and animals. Antibiotics are particularly important to success in these isolations. Identification of the virus types is done mainly in tissue culture at present. These tests are done by mixing known antibody or known virus with either the unknown blood serum or virus and placing the resulting mixture in tubes containing living tissue cells. Most polio viruses cause cells to be destroyed. The presence or absence of virus growth can be read microscopically or by use of dyes indicating metabolic activity.

Tissue culture methods were the means by which Bodian and Horstman first demonstrated that polio virus could spread through the blood stream—a condition known as viremia. This demonstration has discredited the previously accepted theory of the olfactory nerve spread of virus into the central nervous system.

Studies of epidemics and the immunity of the population are readily done by means of tissue culture techniques.

Similarly the search continues for a wonder drug or drugs which might cure this disease, and this is mainly done in tissue culture. Gamma globulin is assayed by means of this technique.

Of all the main avenues of research in the last few years the one most pursued and certainly the most hopeful is the search for preventive or prophylactic methods. Two such possibilities have been tried and are currently being evaluated. These are (1) gamma globulin and (2) preventive vaccines. Each of these should be discussed.

Gamma globulin is one of the blood serum fractions which may be obtained when the liquid portion of blood is split by chemical or physical means. It is in this fraction that the substances are found which inactivate or kill polio virus (and other disease agents). Such substances are found in animals or humans who have encountered and thrown off the disease. Antibody, as this may be called, is capable of being passively, that is artificially, transferred to a second animal or human and still have its protective effect against polio. Bodian first demonstrated this in monkeys. Gamma globulin from humans such as is obtained in Red Cross blood contains appreciable antibody for polio and therefore has been used for prevention of polio. McDowell Hammon of the University of Pittsburgh conducted the first big human trial in 1952. Gamma globulin was compared to an innocuous gelatin solution and found to reduce the incidence of polio in the "G.G." treated children yet the gelatin did not. This type of protection is transient and only lasts 4 to 6 weeks. A gamma globulin injection may reduce the paralytic effects of polio if given one week in advance of the first onset of paralysis but has no effect on the already infected or paralyzed individual. Large pools or mixtures of gamma globulin are found to contain protective substances against all three types of polio.

Attempts have been made to immunize animals and use their serum as a substitute for gamma globulin but this is expensive, difficult, and also only gives the recipient a short time protection.

Closely allied to gamma globulin use is a form of immunization advocated by a few of our outstanding scientists. This is very experimental and involves the injection of "G.G." or immune serum followed in a very short time by the injection of living polio virus. Such

a procedure would provide lasting immunity in the injected person but is dangerous since it would be hard to control the virus that might be excreted by the immunized individual.

By far the most important phase of polio research of the current period is the attempt to produce an efficient and safe vaccine which will prevent the disease. This is not a new endeavor, but the progress made in the last year and that anticipated in the next year has raised the hope of the world. In fact the pace at which this work is progressing can be followed almost daily by picking up almost any newspaper or magazine and reading the feature articles on polio vaccine.

The early vaccines utilized the only available source of virus, namely the brains and spinal cords of infected animals. Monkeys were the main source of these materials. Most of the vaccines contained formalin killed viruses. The actual virus contents of these preparations were low as compared to the current tissue culture virus vaccines and therefore there were failures in immunization. In addition, the heavy protein and tissue content made it difficult to kill the virus. Several instances were reported in which humans were actually infected by such unsafe vaccines. The vaccine which could be made from the brains and cords was quite limited in quantity.

Investigations have been conducted with substances which could boost the protective response of the immunized animal by delaying the absorption of the vaccine. These "adjuvants," as they are called, do accomplish that purpose. Since they are usually of an oily nature they may be dangerous to use. Brain and cord vaccines suspended in such material also have a much more pronounced ability to cause a paralysis—not due to polio—but due to the brain and cord tissue *per se.* The more recent vaccine approaches have been toward eliminating the disadvantages just cited.

In producing a good polio vaccine, or any other virus vaccine, for that matter, the primary need is to obtain the most protection possible per unit of vaccine. A number of methods have been tried in seeking this result.

Adaptation of the polio virus to growth in the embryonated hens' egg has been investigated. Cox has reported on the infection of fertile chicken eggs with a hamster adapted Type 2 virus. His goal is to secure a living virus vaccine which when injected into humans protects them from polio but yet does not cause the vaccinated individual to have the serious disease. The use of such live virus vaccines presents numerous dangers. Also at present only one of the three types of polio viruses can be handled in eggs in this manner.

The advent of practical tissue culture methods for cultivation of viruses provided a ready source of fluids containing very high quanti-

ties of virus. These fluids also had very low protein content. This was a distinct advantage over the old brain vaccines since it made it easier to kill the virus.

Most of you are familiar with the Salk polio vaccine. This is the type of vaccine which has undergone extensive clinical trial during this last year. The so-called "Salk" vaccine is prepared from virus infected tissue culture fluids derived from monkey kidney tissue. Dr. Salk's main contribution to the vaccine program was in developing the methods of killing the virus and making the vaccine safe for human use. Dr. Enders of Harvard earlier had made the first significant start towards such a vaccine when he reported that polio viruses could be grown in tissue culture. A number of laboratories immediately took up use of this method for polio study, many with grants of money from the National Foundation for Infantile Paralysis. The Connaught Medical Research Laboratory of Toronto, Canada, was one of these and they were perhaps the first laboratory to demonstrate that very large quantities of virus could be grown. They placed finely minced monkey kidney tissue in this magical nutrient fluid called M199. After a suitable tissue growth period in a body temperature incubator, they inoculated these kidney cultures with virus and within a few days they obtained a thousand fold increase in virus. Practically all of the material used in the 1954 vaccinations was produced in this manner and much of it by the Connaught Laboratory.

Virus grown in the Connaught or other laboratories had to be made safe for use by killing. Dr. Salk of the University of Pittsburgh worked out the details of this and they involved the use of a combination of heating at 37°C. and the action of a weak solution of formalin. Once the live virus has been converted to dead virus this is then considered as vaccine. Careful methods were instituted which permitted the determination of how long it took to kill each lot of virus. All lots processed have been treated three times as long as it required to just kill the virus. This was done to insure safety in the vaccine.

In order to be effective in the field trials the vaccine had to protect against all three types of virus which caused polio. Therefore, vaccines prepared with each of these three types were mixed. This trivalent or three strain vaccine was then tested for potency and safety before use.

During the period from September 1953 to May 1954 large quantities of this type vaccine were produced under the auspices of the National Foundation. Much of the virus was produced in the Connaught laboratory but some was produced in Dr. Salk's laboratory and in five commercial laboratories, one of these being our own laboratory in Indianapolis. The commercial laboratories, two of us in

particular, did the processing of the virus into vaccine. This was done in close cooperation with Dr. Salk and the National Foundation for Infantile Paralysis. Safety tests were done in three laboratories (1) The commercial firm, (2) Dr. Salk's laboratory, and (3) The National Institutes of Health. The latter group is the governmental agency responsible for safety of such products. Field trials involving thousands of children using these vaccines are now in the process of being evaluated.

At the present time six laboratories are preparing large quantities of vaccine so that when the results of these trials are known there will be large supplies of vaccine available for use in 1955.

Commercial vaccine production on a mass scale may be done in a number of ways. Virus can be produced in several ways. One is the already described method of using minced tissue suspended in rocking cultures of liquid. A second method involves the growth of cell sheets on the inside surfaces of glass bottles. This may be accomplished by treating monkey kidney tissue with trypsin, stopping the enzyme action, and then bathing these cells with a nutrient medium which permits the cells to adhere to the side of the glass container and grow. Similarly pure cell strains such as the HeLa epithelial cell strain may eventually be used for large scale production. With the exception of cell strains the methods involve the use of large numbers of monkeys as a source of normal kidney tissues. Many laboratories such as our own have had to learn how to handle large monkey quarters. Several of these will each house several thousand monkeys at one time.

Most large scale polio virus production is done in bottles which range in size from one pint to approximately five quarts; however, only a small part of each bottle is actually full of fluid for growing virus. Two growth phases must occur before virus is obtained. The tissue is first cultivated for four to ten days and then the virus is added along with new nutrient fluids. The virus harvest is made three to five days after the tissues are infected with virus. The virus fluids from a number of such bottles are pooled and this is filtered to remove tissue and any possible contaminating bacteria or fungi. The polio filtrations now being done routinely are the first time that viruses have been successfully filtered on a large scale.

Filtered virus fluids are heat and formalin killed as previously described. Other killing agents such as ultraviolet, nitrogen mustards, etc. may eventually replace heat and formalin but all commercial vaccine is now formalin killed.

After the viruses are killed the three types are combined as a polyvalent vaccine and this is then tested in monkeys for safety and potency. Another test for the inactivation of virus is simultaneously made by putting the vaccine back into tissue cultures. Any residual

living virus would then infect the cells and destroy them. The safety test monkeys are observed for symptoms of polio and after twenty-eight days sacrificed and the brains and spinal cords examined for polio damage. Any vaccine which produces any symptoms or lesions in either monkeys or tissue culture would be considered unfit for human use.

Potency tests on the vaccine consist of the injection of monkeys and mice with the vaccine and then testing the blood of these animals for antibody to the three types of virus. Methods of quantitating this response are in the process of being worked out at the present time. Data from mice and monkeys indicate that the vaccines used in the clinical trial and currently being produced are effective in producing an antibody response. This is certainly an indication that the vaccine has a potential of protecting animals and people.

The human data on polio vaccination is only partially complete. Antibody has been produced in humans to a relative high titer and this is very good news, however, it will be some months before we know whether the vaccine has protected people from infection. Most people are optimistic since the infection probably spreads through the blood stream in most cases. However, if the virus is spread through the body by nerve cells the protection may not be as good or perhaps not even be demonstrable.

Two notable advances have been made in the last year other than the Salk vaccine. One of these is the first electromicroscopic examination of the virus. The use of this method is new to polio and will aid materially in purification studies. The other advance is the application to genetic principles to the polio viruses in such a way as to produce strains of greatly altered infectivity. These genetic variants of the polio viruses may change the entire approach to vaccine and current methods of research.

Up until now we have been discussing either the past or present work in this field. Perhaps a glance into the future is warranted. Prophylaxis or prevention of polio looks to be very hopeful and better vaccines will soon be found. Therapy has improved but much progress still may be accomplished. The method of dissemination of the disease and other epidemiological studies remain to be enlarged upon in order to help prevent the disease.

Much of the knowledge obtained in polio research is applicable to other diseases. In fact is has been stated that the cure for cancer might be a result of polio research and conversely the polio problem might be solved by cancer research. Actually this last is partially true already since tissue culture methods were really evolved for cancer work.

One surely might conclude with the statement that the last five

years of polio research and development has been one of the greatest in the medical and scientific history. The next few years may be even more productive.

MATHEMATICS IN ELEMENTARY PHOTOGRAPHY

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It is not often that a teacher is privileged to teach a course in photography since it is not often included in the high school curriculum, nor is it easy to get a teacher with the proper training. However, photography offers a very good medium for instruction in certain principles of chemistry, and a marvelous opportunity for creative teaching of topics in physics and mathematics. It is the purpose of this paper to show some examples of how photography makes use of applied mathematics.

I. Box Camera Mathematics

The student who shoots a roll of film with his box camera, takes the exposed roll to the corner drugstore, and gets the prints back a day or two later is a good subject for a lesson in "cost accounting." A roll of film usually contains 8 pictures. Let us assume they all come out. The problem: how much do you pay per picture?

Discussion: A roll of 120 film costs about $.46. Developing is usually $.20 with a charge of $.07 per print. The total cost is then $.46+.20 +8(.07) = $1.22. The cost per print is $1.22÷8 or about $.15.

Problem: Devise a formula for the cost of developing and printnig any roll of film.

Solution:

$$C = P + .20 + .07N$$

where

C = total cost

P = price of film

N = number of prints

Problem: Draw a graph which relates the cost of purchase, developing, and printing a roll of 120 film.

Solution: See Figure 1. Incidentally, it should be noted that the straight line connecting the points is actually meaningless (since you cannot have a fraction of a print) but it does serve to show the linearity of the relationship. The same applies to Figure 2.

Problem: Devise a formula which gives the cost per print. Graph

this formula.

Solution:

$$C = \frac{P + .20 + .07N}{N} .$$

See Figure 2.

The foregoing examples can be varied in many ways and extended to problems in costs of paper and chemicals, and discount problems on cameras, enlargers, and other equipment. An interesting study might be made on the amount of depreciation of new equipment when traded in or bought used. Information for these problems can be obtained from catalogs, photo magazines, or the local camera store.

II. ELEMENTARY PHOTOGRAPHIC OPTICS

For a given level of illumination the amount of light entering a camera depends on the area of the lens opening. The amount of light falling on the film (which is actually responsible for the image formation) is related directly to the lens opening and inversely to the distance from lens to film. In the interests of uniformity the f-number system of lens marking has been adopted to relate the factors of amount of light falling on the film, lens opening, and lens-to-film distance.

$$\text{f-number} = \frac{\text{lens-to-film distance}}{\text{diameter of lens opening}} .$$

The lens-to-film distance, which is nearly constant for most cameras, is for most photographic work the same as the focal length of the lens. In all but the simplest cameras the diameter of the lens opening can be varied by means of an iris diaphragm. Since the focal length of the lens is fixed the f-number is inversely proportional to the diameter, a small lens opening being indicated by a large number. For example, f/8 admits twice as much light as f/11.

The calculation of f-numbers is an interesting exercise in squares, square roots, and elementary geometry. It was noted above that f/8 gives double the light transmission of f/11, yet 8 and 11 are neither halves nor doubles of each other. Let us use the formula to find the actual diameters of the openings (assume any focal length—say 5 inches).

$$f/8 = \frac{5''}{D} \qquad\qquad f/11 = \frac{5''}{D'}$$

$$D = \frac{5}{8} = .63'' \qquad\qquad D' = \frac{5}{11} = .45'' .$$

But D is not double D'. How then can we say we have doubled the light transmission? The answer lies in the fact that the light transmission depends on the area of the lens opening, not its diameter. If we use the theorem that areas of circles are proportional to the squares of the diameters:

$$\frac{A}{A'} = \frac{.63}{.45} = \frac{.40}{.20} = \frac{2}{1} .$$

We see, then, that one area is actually double the other. If one f-number of a lens is known, the others can be calculated by simply squaring the f-number, halving or doubling it, and taking the square root; or else by multiplying or dividing the f-number by the square root of 2 or 1.415.

III. Projection Systems

While the design of projection equipment is a matter for experts, the basic theory of image projection is relatively simple. A beam of light is passed through a transparent film or slide, then through a lens and onto a screen. The basic lens formula

$$\frac{1}{\text{Focal length}} = \frac{1}{\text{Object distance}} + \frac{1}{\text{Image distance}}$$

applies in this case, as it does also to camera lenses. In practice a condensing system is added to concentrate the light and the lens-to-film distance may be varied to allow for various projection distances (or "throws"). As the projector is moved closer to the screen the image size will of course decrease, and the lens-to-film distance increases. This can be shown by the above formula. Since the focal length is constant, if the image distance decreases, the object distance must increase.

We can compute the size of a projected image by similar triangles. In essence, a projection system resembles Figure 3. Note that the Object (film or slide), Image, and the light rays form a pair of similar triangles. Therefore,

$$\frac{\text{Object}}{\text{Object distance}} = \frac{\text{Image}}{\text{Image distance}} .$$

For example, in a 15 foot living room a 35 mm. slide (film dimensions about 1 by $1\frac{1}{2}$ inches) projected with a 4 inch projection lens will give an image about 45 by 67 inches.

IV. Flash Photography

When a subject is close to a flashbulb it is illuminated by an intense

light as the bulb is fired. As the distance from flashbulb to subject increases, the light intensity becomes less. The lens must be opened or closed to compensate for these differences. To calculate the proper lens opening we use a guide number which can be obtained from the table which appears on each package of flashbulbs. This guide number depends on the type of flashbulb and film and to some extent by the shape and finish of the flashgun reflector. For example, if a number 5 flashbulb is used with ordinary black-and-white film, the guide number is about 160. The lens setting is obtained by dividing the guide number by the distance from flashbulb to subject. The quotient is the lens setting.

Example:

$$\text{for 10 feet, } 160 \div 10 = f/16$$

$$\text{for 20 feet, } 160 \div 20 = f/8.$$

V. DARKROOM MATHEMATICS

If students are given an opportunity to work in the darkroom, the range of uses of mathematics is extended.

Problem: How long should a given type of film be developed, given the developer and its temperature?

Solution: This will involve reading a time-temperature development graph such as the one in Figure 4. A series of these graphs for many types of film is given in the Eastman Kodak Data Book on *Films.*

Of course many problems can be devised on dilution of solutions, conversion of units from metric to English, making only half (or one-fourth) a given formula, etc.

VI. SUMMARY

It is hoped that the examples given in this discussion will stimulate teachers of photography to include some mathematics in their courses, since teaching mathematics with photography has a double advantage: it is easily motivated, and the subject is recalled whenever the camera is used.

The author wishes to thank the Sales Service Division, Eastman Kodak Company, for their help in the preparation of this article.

DEMONSTRATIONS WITH RADIOISOTOPES FOR THE HIGH SCHOOL CHEMISTRY CLASS*

GEORGE J. GOLDSMITH

Purdue University, Lafayette, Ind.

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    #52 *Maximum Permissible Amounts of Radioisotopes in the Human Body and Maximum Permissible Concentrations in Air and Water.*
    #51 *Radiological Monitoring Methods and Instruments.*
    (These are available from the U. S. Gov't. Printing Office for 20¢ each.)

AVAILABILITY OF RADIOACTIVE MATERIALS

In general, the procurement of radioactive isotopes from any source comes under the regulations of the United States Atomic Energy Commission except for certain naturally radioactive materials such

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† These experiments will be demonstrated.

as radioactive watch dials and small quantities of uranium, thorium, and radiolead.

The procurement of artificially radioactive materials from the Atomic Energy Commission requires the approval of an application form on which one must indicate that he has had suitable training and has adequate facilities to handle these substances. It is not likely that a high school science laboratory will meet the requirements of this application.

However, there are provisions in the AEC regulations for supplying small quantities of materials normally requiring authorization without this authorization. These are so called exempt quantities. The following is a quotation from the Code of Federal Regulations: Radioisotope Distribution, Title 10—Atomic Energy; Part 30—Radioisotope Distribution, covering these exemptions:

30.13 *Items and Quantities.* (a) Sections 30.20 through 30.61 inclusive, do not apply to any item listed in Section 30.70 (Schedule A) nor to any quantity listed in Section 30.71 (Schedule B) provided, however that no person shall, except as otherwise permitted by the regulations contained in this Part, effect an increase in the radioactivity of such scheduled items or quantities by adding other radioactive material thereto, by combining the radioisotopes from two or more such items or quantities, or by altering them in any other manner so as to increase thereby the rate or radiation exposure of himself or others above the original rate therefrom.

(b) In addition the Commission may, upon application of any interested party, exempt specific items from the application of all or any portion of the regulations in this part subject to such conditions as the Commission may establish whenever the Commission determines that the possession, use, or transfer, of such items will not endanger health or present a hazard to life or property.

30.70 *Schedule A:* Exempt items. (See Section 30.13.) None.

30.71 *Schedule B:* Exempt quantities (See Section 30.13).

    (a) Alpha Emitters: None.

    (b) Beta and Gamma Emitters: Not more than a combined total of *0.011 millicurie*, made up as follows: (1) *Half-lives no greater than 30 days: Not more than 0.010 millicurie.* (2) *Half-lives greater than 30 days: Not more than 0.001 millicurie.*

    (c) Neutron Emitters: None.

    NOTE: The quantities listed in Schedule B are not to be interpreted or considered as having any bearing on the determination of safe permissible levels of personnel exposure or for waste

disposal. It is the Commission's intention to publish at a later date an incorporate in this part appropriate health and safety standards.

At the present time the AEC does not supply exempt quantities directly, though the matter is under consideration. For those who are interested in keeping up to date on the status of the consideration, information may be obtained from:

> U. S. Atomic Energy Commission
> Isotopes Division
> Post Office Box E
> Oak Ridge, Tennessee

To our knowledge, there are two suppliers of exempt quantities outside the AEC, they are the following:

> Fisher Scientific Company

> Tracerlab, Incorporated

It is possible that there are others, but they have not come to our attention.

Two sources of naturally radioactive lead and its daughter products are the following:

> Hammer Laboratories
> 8139 Lemon Avenue
> Le Mesa, California

> Canadian Radium and Uranium Corporation
> 630 Fifth Avenue
> New York 20, N. Y.

THE GEIGER COUNTER

REFERENCES

Korff, S. A., *Electron and Nuclear Counters*. New York: D. Van Nostrand Co., Inc , 1946.

Bleuler, E., and G J. Goldsmith, *Experimental Nucleonics*. New York: Rinehart and Co., Inc., 1952. pp. 49–79.

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Pollard, Ernest and William L. Davidson, *Applied Nuclear Physics*. New York: John Wiley and Son., Inc., ed. 2, 1951. pp. 31–37.

Whitehouse, W. J. and J. L. Putnam, *Radioactive Isotopes*. London: Oxford University Press, Amen House, 1953. pp. 146–159 and 161–170.

Lapp, Ralph E. and Howard L. Andrews, *Nuclear Radiation Physics*. New York: Prentice Hall, Inc., 1954. pp. 226–234.

The Geiger counter consists of a thin wire anode mounted along the axis of a hollow conducting cylinder, the walls of which make up the cathode. The cylinder is generally filled with a rare gas at reduced

pressure along with a few per cent of an organic vapor. An electric potential of the order of 1000V is applied between the central wire and the cylinder wall setting up a field which becomes increasingly intense toward the region of the central wire. When ionizing radiation enters the sensitive region of the Geiger counter and produces at least one ion pair, the electron will be accelerated toward the positive central wire. In the process of acceleration, this electron will collide with gas molecules producing more ionized pairs which in turn will make further collisions and produce eventually an avalanche of ionization along the central wire. This results in an enormous amplification of the initial ionization produced in the counter. The enhancement of the ionization is so great that the total charge released in the avalanche is essentially independent of the initial amount of ionization. The end result of this is a voltage pulse, the amplitude of which does not depend upon the initial ionization.

In the gas mixture the electrons have a very high velocity while the positive ions move quite slowly. Therefore, after the electron avalanche is collected on the central wire, there remains behind a sheath of positive ions which are migrating relatively slowly toward the cathode. This ion sheath shields the central wire in such a way that the electric field near it is very low and hence, during the time that the sheath is moving outward, a new avalanche cannot be initiated and the counter is insensitive. This period of insensitivity, called a deadtime, is the principle limitation on the maximum counting rate at which one can use a Geiger counter.

Types of Geiger Counters and Scalers

Geiger Counters:

Basically, all Geiger counters are of the same design consisting of a conducting cylinder surrounding a thin axial anode. Except for specialized instruments such as the Libby screen-wall counter, the various modifications differ in the manner in which the radiation is permitted to enter the sensitive region and in the filling gases. The Geiger counter has been developed to such an extent that practically every conceivable variation of gases and methods for admitting radiation have been employed. The important differences which should be observed in this demonstration are the specific applications for the various types and their range of versatility.

Scalers:

The scaler is essentially a computer which receives information from the counter and converts it into a form which can be recorded. The problem which the scaler must solve is the following:

Radioactive sources which are of sufficient activity to permit one

to measure them with high precision in a short period of time must deliver their information at a rate far too large to permit any mechanical device to record it accurately. The scaler takes the information which is being transmitted to it as this high rate and divides it by a constant factor yielding a rate which can be recorded mechanically.

The types of scalers differ only in the manner in which they accomplish the above-mentioned task. The basic modifications are in the scaling system, whether it is a binary system or a decade system, and in the manner in which the information is recorded. Since all counting is a rate measurement, one may measure either the total number of counts in a given period of time, or the time necessary to record a given number of counts. Thus, there are essentially three scaling systems:

a. Those counters which will record the total number of counts received divided by the scale factor on a mechanical register for manual determination of counting rate.

b. Those which will count automatically for a selected period of time.

c. Those which will record the time necessary to accumulate a selected number of counts.

Some scalers have provisions for all three systems. There are also more elaborate modifications in which the entire counting operation is made automatic by having a mechanical device which changes the sample, resets the scaler, and records the data on a printed paper tape.

N-GAMMA ACTIVATION—RADIOACTIVE DECAY

REFERENCES

Bleuler, E., and G. J. Goldsmith, *Experimental Nucleonics*. New York: Rinehart and Co., Inc., 1952. pp. 105–116.

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Mattauch, J. and S. Fluegge, *Nuclear Physics Table and Introduction to Nuclear Physics*. New York. Interscience Publishers, Inc. 1949. pp. 65–93.

In this demonstration two problems associated with radioactivity will be studied. The first is the production of radioactive isotopes and the second, radioactive decay.

Nuclear transmutations are produced by the interaction of either

energetic charged particles, energetic photons, or neutrons with the nuclei of matter. The sources of high energy particles are the large machines (cyclotron, synchrotron, linear accelerator, betatron, etc.) while neutrons are produced in a nuclear reactor (pile) or as products of other nuclear reactions.

In this demonstration metallic silver which has two stable isotopes of mass numbers 107 and 109 will be irradiated with thermal neutrons. Neutron capture results in the production of two new silver isotopes (mass numbers 108 and 110)both of which are radioactive. After the production of these radioactive isotopes a study will be made of the nature of radioactive decay by measuring the decay rate.

The neutrons which are used in this experiment are produced by the nuclear reaction $Be^9(\alpha, n)C^{12}$. A small quantity of radium serves as the source of alpha particles. It is sealed together with some beryllium powder in a nickel capsule.

This reaction results in neutrons which are far too energetic to produce transmutations efficiently. The neutrons are therefore slowed down to thermal energies through collisions with hydrogen nuclei contained in paraffin which surrounds the source. One can consider that the paraffin blocks become filled with a neutron gas which has greatest density near the source. The material to be irradiated,in this case silver foil, is placed inside the paraffin close to the source. Consideration of the radiation time will be made following the discussion of radioactive decay.

The radioactive isotopes which are produced in this manner (silver 108, and 110) decay through the emission of a beta particle converting the silver to stable Cd (isotopes, Cd^{108} Cd^{110}). The energy of the emitted beta particles and the rate at which they are given off, is characteristic of the particular isotope in question, following definite laws. The radioactive decoy obeys a statistical law, identical with that encountered in monomolecular chemical reactions. That is, the rate of decay is proportional to the total number of radioactive atoms present. This results in a decay rate which decreases exponentially. Expressed mathematically this becomes:

$$\frac{dn}{dt} = -\lambda n$$

where

$n = $ number of atoms present

$t = $ time

$\lambda = $ the decay constant.

The solution of this equation is:

$$n = N_0 e^{-\lambda t}$$

where n is the number of atoms initially present. This means that the intensity of radiation, I, may be expressed as

$$I = I_0 e^{-\lambda t}$$

or in another form

$$\ln \frac{I}{I_0} = -\lambda t.$$

If one plots then, the logarithm of the relative counting rate, versus the time, a straight line function results. The time at which the activity is decreased to one half its original value is significant. This is the characteristic half-life of the material. It is interesting to note that this half-life is completely independent of any chemical form or chemical reaction to which the material is subjected. It is a characteristic only of the nuclear state.

The half-life of the isotopes which is being produced in a nuclear reaction has a definite bearing on the total time over which one can efficiently produce transmutations. Since the rate of decay depends upon only the number of radioactive atoms present, then as one produces transmutations, the rate of decay of the material will gradually increase. The number of radioactive isotopes will cease to increase at the time when the rate of decay equals the rate of production. There are two cases to consider. If R is the rate of production at any instant by the source, the increment of active atoms is

$$\frac{dN}{dt} = R - \lambda N - \frac{1}{\lambda} \ln (R - \lambda N) = t + C.$$

Since $N = 0$ when $t = 0$,

$$C = \frac{1}{\lambda} \ln R$$

and

$$-\lambda t = \ln \frac{R - \lambda N}{R}$$

Solving for N, one obtains

$$N = \frac{R}{\lambda} (1 - e^{-\lambda t})$$

or with

$$= \frac{0.693}{T_{1/2}}$$

$$N = \frac{RT_{1/2}}{0.693}\left(1 - \frac{-0.693t}{eT_{1/2}}\right)$$

which is sometimes written more conveniently

$$N = \frac{RT_{1/2}}{0.693}\left(1 - 2\ \frac{-t}{T_{1/2}}\right).$$

From this it can be seen that for an irradiation time $t = T_{1/2}$, 50% saturation is reached, and for $t = 2T_{1/2}$, 75% saturation is reached; bombardment times greater than three or four half lives give very little further increase in the activity. On the other hand, in the production of radionuclides having half lives very long relative to the bombardment time, the amount of activity increases nearly linearly with bombardment.

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CONTINUOUS DIFFUSION-TYPE CLOUD CHAMBER

NOTES ON CONSTRUCTION
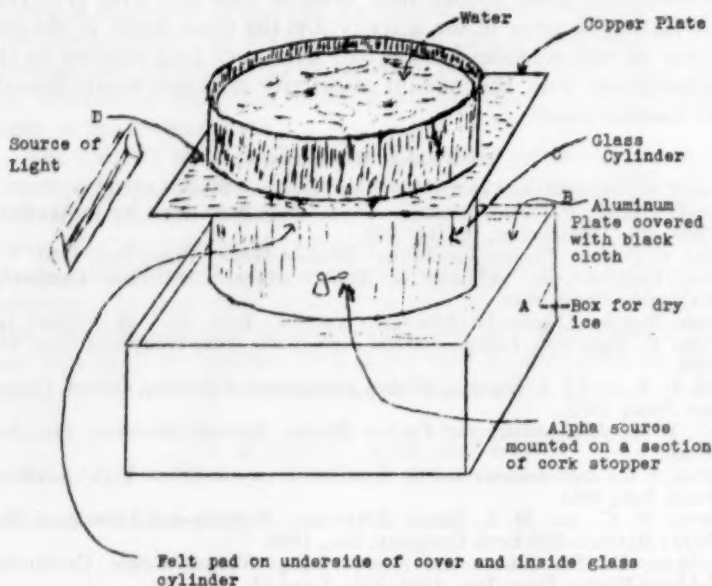
The container (A) for the dry-ice is made of wood base with masonite sides (6.5"×6.5"×2.5") interior dimension. Plates of dry ice 6"×6" by any desired thickness are used. Plates of corrugated cardboard cut 6"×6" serve to hold the top surface of the dry-ice just under the top edge of the container. The thickness of the dry-ice will be controlled by the period of time the cloud chamber is to be in operation.

A plate of aluminum (B) about $\frac{1}{16}$" thick is cut to fit the interior of the box. The top surface of this plate is covered with black velvet or black felt. This plate is placed on the top surface of the dry-ice slab If the condensation of water vapor from the atmosphere has formed

an ice layer over the dry-ice this should be removed before placing the aluminum plate in position. The aluminum plate should make good thermal contact with the dry-ice.

A glass cylinder (C) $5\frac{1}{2}$" in diameter and $3\frac{1}{2}$" long is cut from a 3000 ml. Pyrex beaker. This is placed on the cloth covered aluminum plate as shown.

The cover (D) is made by soldering a strip of copper to a copper plate to form a cylindrical tank $6\frac{1}{2}$" in diameter and 2" deep. A plate of felt $\frac{3}{4}$" thick and 5" in diameter (cut from an old felt boot or other source) is fastened to the center of the underside of the copper cover by wires soldered to the cover.

In operation the felt pad is saturated with isopropyl aclohol (rubbing alcohol) and the tank on the cover is filled with water at room temperature. The water serves to keep the alcohol-soaked-pad at near room temperature thus setting up an appropriate temperature gradient within the chamber to establish conditions for cloud formation.

The alpha source is the luminous hand of a clock, the type where luminosity is due to the presence of a minute amount of a radium salt.

Side illumination using a fluorescent desk light or projector causes the cloud tracks of the alpha particles to be readily visible.

The clearing field is formed by using a piece of Lucite $8" \times 8" \times \frac{3}{16}"$. This is charged by rubbing with a piece of cats fur or a woolen cloth and placed on top the water tank on the cover. This induces a charge on the metal plates and establishes an electric field within the chamber which will remove the residual ions.

This chamber might be used to observe tracks of cosmic rays and beta particles, however the density of ionization is so small as to make these tracks difficult to observe.

The dimensions given here are taken from a cloud chamber which works well. Deviations in dimensions would be possible.

LOWEST DEATH RATE, MOST BABIES IN '54

A record number of new Americans, 4,000,000 of them, arrived in 1954. Along with this bumper baby crop, the year set another record: the lowest death rate in the nation's history. The figures were announced by Dr. Leonard A. Scheele, Surgeon General of the Public Health Service of the U. S. Department of Health, Education, and Welfare, on the basis of vital statistics reports for the first 10 months of the year.

The marriage rate declined to 9.2 per 1,000 population. Low birth rates in the depression 1930's are responsible.

Divorces were also on the decline, according to the figures for the first nine months of 1954. Since the 1946 peak, these rates have dropped over 40%.

THE PROOF BY NINE*

(La prueve par neuf)

JOHN G. GOSSELINK
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In parts of Europe, particularly the French speaking countries, a method for checking multipication and division is used, that is so simple that it might be advantageous to adopt this method, widely, in this country. It is there taught in the public schools, almost as soon as the process of multiplication is taught, consequently they regard it as intergal part of the process. Those who have been taught this process in their youth would feel that they had left the operation half completed, if they did not subject the result to this simple check. Also, the method contains many intriguing features that will interest those who like mathematics. Besides this, it is an aid to accuracy, so the interest in this process, used by the French, should be general.

The general procedure used is to drop all nines, and to add the digits of numbers larger than nine. How this applies to the process in general will become apparent in the following illustrative example.

Suppose, to chose two numbers at random, we use 6549 as the multiplicand and 7546 as the multiplier. The result of the multiplication is 49418754.

```
        6 5 4 9
        7 5 4 6
      ─────────
        3 9 2 9 4
      2 6 1 9 6
    3 2 7 4 5
  4 5 8 4 3
  ─────────────
  4 9 4 1 8 7 5 4
```

To check, we make a cross beside our multiplication.

First—we add the digits of the multiplicand by the rules of the general procedure. $6+5=11$, which is larger than nine, so we add the digits of the 11 obtained $1+1=2$, then continue to the next digit which is 4, $2+4=6$, which is the figure we are after, since the final figure is 9, which by the general procedure must be dropped, not added. We place the 6, thus obtained, in the upper part of the cross.

Second—We apply the same process to the multiplier. $7+5=12$, which is larger than nine, so we add the digits of the 12 obtained, $1+2=3$, then continue to the next digit which is 4, $3+4=7$, this is

smaller than nine so we continue to the last digit 6, $7+6=13$, which is larger than nine so we add the digits of the 13 obtained, $1+3=4$, which is the figure we are after. We place this digit, 4, in the bottom of the cross.

Third.—We multiply the digit in the top of the cross by the digit in the bottom of the cross. $6 \times 4 = 24$, which is larger than nine, so we add the digits of the 24 obtained, $2+4=6$, which is the figure we are after. We place this digit, 6, in the right hand side of the cross.

Fourth.—We add the digits of the answer, by the same procedure. The second digit of the answer is a nine, so we simply drop it. We thus add the first and third digits, $4+4=8$, this plus the next digit, 1, equals nine, so we again drop the nine, continue to the next two digits, 8 and 7, add them, $8+7=15$, which is larger than nine, so we add the digits of the 15 obtained, $1+5=6$, continue to the next digit 5, add, $6+5=11$, which is larger than nine, so we add the digits of the 11 obtained, $1+1=2$, which plus the final digit 4, equals 6, the figure we are after. We place this digit in the left hand side of the cross. If the two digits on the opposite sides of the cross are the same, this means the problem is proved, the multiplication, as performed, is correct.

In going through this process step by step as we have done, including each detail on a rather long multiplication, this method does not seem to be a great improvement over repeating the multiplication. In actual practice, after one has become familiar with the process it can be performed very rapidly. This is because various short cuts, that will be discussed later, help improve this method of addition, making it very rapid. Try the method on a few multiplications of your own, to become more familiar with the process. It is remarkable how it checks out every time, isn't it?

What makes the method work? Black magic? Does this dropping of the nines have a supernatural influence? Except for the few, who are greatly intrigued by mathematical enigma, who may wish to stop reading at this point to see if they can puzzle this out for themselves, the rest of us will want to know what makes the method "tick."

First—let us dispell the notion, that there is anything of a magical nature associated with the dropping of the nines, as might appear both from the nature of the process itself, and from the nomenclature associated with it. The dropping of the nines is simply a short cut, within the short cut of the main process. It is fairly obvious, that if we add nine to any number, using the above method of adding, we will always get back the original number. Take 69 as an example. $6+9=15$, this is larger than nine, so we add the digits of the 15 obtained, $1+5=6$, we get exactly the same digit, that accompanied the nine in the first place. Instead of going through the addition to obtain this 6, it is much simpler to drop the nine in the first place, and retain

the 6. Try this on the answer to our multiplication above, which contains a nine, and you will find this is so.

In fact, several variations, in the manner of adding are possible. For instance, we may add all the digits to one total, continue adding this total till we obtain one digit, and the same result will be achieved. Take the answer to our problem, given above, as an example, $4+9+4+1+8+7+5+4=42$, $4+2=6$, we get the same digit 6, that we got when we dropped the nines. For some, who rarely make a mistake in addition, this may, at least until they become accustomed to the special manner of adding, seem simpler than dropping the nines. If the 42 obtained by direct addition is divided by nine, the remainder, 6, will be the number we are after. Division by nine eliminates all the nines in the sum, thus the remainder is the digit desired. Perhaps the most rapid, the most accurate method, the method we prefer, is to eliminate all obvious combinations that give nine, by inspection, and cancellation, and then add by the method previously developed the remaining digits. Again we take our answer 494187544 as an example. *494187544*. This contains a nine, so we immediately strike it out. Then we immediately see, by inspection that $4+4+1=9$, so we strike out those three digits. Then we see, by inspection the final two digits, $5+4=9$, so we strike out these two digits. This leaves to be added, $8+7=15$, which is larger than nine, so again we add, $1+5=6$ and achieve the same result.

Thus we see, there is no magical quality, imparted to the method by the dropping of the nines, for the method works without it. The dropping of the nines is simply a shortcut, within the framework of the main method. (If there are any who stopped reading at the point we suggested it, to work out the solution for themselves, gave up in disgust, misled by the apparent effect of dropping the nines, they may take a fresh start at this point if they wish.)

It is obvious, that if you make a set of multiplication tables extensive enough (going beyond the 12 tables taught to us in the grades) that any two numbers you may wish to multiply together, will be found in these tables. The real reason the method works, lies in the fact, that if you prepare a multiplication table of any number whatsoever, however large, as you proceed along the table, if you subject the multiplicand and multipliers to the same process that is used to obtain the digit on the right hand side of the cross, you will find that the digits obtained, as you proceed along the table, form a series, that endlessly, over and over again, repeats itself.

If however, as you proceed along the table, you subject the answers found in your multiplication table, to the same process that is used to obtain the digit on the left hand side of the cross, you also get a series, that endlessly, over and over again, repeats itself.

These series, though derived by two different methods, are identical in every respect, member for member.

Thus it makes no difference in the series number obtained, whether you add the digits of your multiplicand, and of your multiplier first, then cross multiply, or whether you first multiply, by the usual method, and then add the digits of your answer. In each case you will get a series number, characteristic of the particular multiplication performed, arrived at by two independent methods. Thus the digits that appear on the right and left hand side of the cross, are series numbers, characteristic of the particular multiplication, and only if the multiplication has been done correctly will the two digits agree.

That a multiplication table gives such a series is most readily illustrated by the nine table, which gives the simplest series of all. No matter what number you multiply by nine, if you subject the answer to addition to a single digit, by the rule given above, the series number obtained will always be nine. Thus the series formed by the nine table is 9, 9, 9, etc. endlessly repeated. For other multiplication tables, the series that result are in general not so simple, but in every case, the series will be endlessly repeated.

Below, as an example, we have constructed a table of fourteens, in

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7
1	1	14	5	5	14	5
2	2	14	5	1	28	1
3	3	14	5	6	42	6
4	4	14	5	2	56	2
5	5	14	5	7	70	7
6	6	14	5	3	84	3
7	7	14	5	8	98	8
8	8	14	5	4	112	4
9	9	14	5	9	126	9
10	1	14	5	5	140	5
11	2	14	5	1	154	1
12	3	14	5	6	168	6
13	4	14	5	2	182	2
14	5	14	5	7	196	7

which the series numbers appear, which will make the above discussion much clearer. The series developed by the fourteen table is 5, 1, 6, 2, 7, 3, 8, 4, 9, etc., endlessly repeated. In column 5 of this table the series was arrived at, by adding the digits of the multiplier and multiplicand separately, cross multiplying, and continuing adding to a single digit when necessary. In the column at the extreme right, Col. 7, the series was arrived at by adding the digits of the answers to a single digit, following the method developed previously. Thus the series numbers that develop in Col. 5 will be the series number

that would appear on the right hand side of the cross, if we checked the separate multiplications used to construct the table, by the Proof by Nine. The series numbers that appear in column 7 is the series number that would appear on the left hand side of the cross.

The explanation of what each column contains is given below the table, and should be read, as the table is examined.

Col. 1 gives the multipliers. Col. 2 gives the sum of the digits of the multipliers, added when necessary to obtain a single digit. Col. 3 gives the multiplicand. Col. 4 gives the sum of the digits of the multiplicand, added by the special method used in this process till a single digit is obtained. Col. 5 contains the numbers found by multiplying the number found in Col. 2, by the corresponding number in Col. 4, and the individual digits of this answer added to obtain a single digit. Note that in this part of the table we have performed exactly the same operations that were used to obtain the digit in the right hand side of the cross.

Col. 6 gives the answers to the multiplication table. Col. 7 gives the result when the digits of the answers are added to obtain a single digit. Note, that in this part of the table we have used exactly the same operation we used to obtain the digit found in the left hand side of the cross.

Col. 5 agrees in every detail with Col. 7, member for member, and though the series obtained for other multiplication tables, will be different the two columns will always agree, in any multiplication table, member for member, endlessly repeating themselves. This is what makes the method work. What we are actually doing is comparing the series number that applies to that particular multiplication, as it appears on the right hand and left hand side of the cross; these have been obtained by two independent methods, and only if the multiplication was correctly performed will they agree.

Since this method is based on sound mathematical principles, is the method infallible? Unfortunately, "No," though this answer will undoubtedly sound like rank heresy, to many Frenchmen, who have used the method all their lives, and have yet to find a case where it failed, and have thus come to regard this method as a mathematical gospel.

Nevertheless, the method can fail, if for want of a better term, "compensating errors" are made. Let us see how such a "compensating error" might develop. The answer, to our problem above, was 49418754. But if in working the problem, we make two errors, either in multiplying or adding, in which say the third digit becomes too large by one, and say the sixth digit becomes too small by one, then the answer we would have obtained would have been 49518654. This

too would give the series number six, by addition, and by the method of nines the error would not be detected.

But to make an error of this sort, in which the errors exactly compensate becomes very rare in actual practice, especially if the problem is shorter than the one we have chosen. We believe it is much safer, for instance, than going over the multiplication figure a second time, a common practice. With the figures before you, subconsiously suggesting the figures you obtained before, one is apt to make the same error over and over again, even when you know an error is present. The French speaking race would scarcely teach this method in their schools, if experience had taught them that the method was continually failing. The opposite is apparently true, to most of them it would come as a shock to learn that the method was not infallible.

The real check for multiplication of course is division, and if a problem is really important, this method should be used. Even here, we suppose it is theoretically possible to make the same errors in reverse order, but the chances that this would happen become almost infinitesimal. Even when division is used, this method gives a quick, additional check.

It would be unfair to present this method without stipulating its limitations; it should not be taught as absolutely infallible as is too often done by the French.

Now let us see how the method is used to prove division.

Suppose we take the problem:

```
   2 7 3 / 3 5 7 8 6 / 1 3 1
         2 7 3
         ─────
         8 4 8
         8 1 9
         ─────              3
         2 9 6          2  ╳  2
         2 7 3              5
         ─────
           2 3
```

First.—We add the divisor, $2+7=9$, drop it since it is a nine, leaving the digit 3. Put the result in the top of the cross.

Second.—We add the answer to the problem, $1+3=4$, $4+1=5$. Put the result in the bottom of the cross.

Third.—Multiply the top of the cross by the bottom, $3\times5=15$, since this is larger than nine, add, $1+5=6$, to this add the remainder, if a remainder has developed in the problem, $6+2=8$, continue, $8+3=11$, and since this is larger than nine, add, $1+1=2$. Place this in the right hand side of the cross.

Finally.—Add the number divided, $3+5=8$, $8+7=15$, $1+5=6$, $6+8=14$, $1+4=5$, $5+6=11$, $1+1=2$. Place this in the left hand

side of the cross. If the two sides of the cross check, the problem is proved.

The same basic principle, which causes the right and left sides of the cross to check in multiplication, also operates in division, except that an extra step is taken in division. This is, that the remainder is added to the series number on both sides of the cross in division, though it is not immediately apparent that it has been added to the left side. This will become clear if we perform our division as follows:

$$\frac{2\ 7\ 3\ /\ 3\ 5\ 7\ 6\ 3 + 23\ /\ 1\ 3\ 1}{2\ 7\ 3}$$
$$\frac{8\ 4\ 6}{8\ 1\ 9}$$
$$\frac{2\ 7\ 3}{2\ 7\ 3 + 23}$$

35763+23 equals our dividend 35786. But now, for the moment neglecting the remainder the division comes out even. 273 divided into 35763 equals 131, and conversely, 273 times 131 equals 35763.

If we follow the individual instructions, given above, for this multiplication, and this division, we find that we are going through the identical operation, in checking these two problems.

$$\frac{2\ 7\ 3\ /\ 3\ 5\ 7\ 6\ 3\ /\ 1\ 3\ 1}{2\ 7\ 3}$$
$$\frac{8\ 4\ 6}{8\ 1\ 9}$$
$$\frac{2\ 7\ 3}{2\ 7\ 3}$$

$$6 \underset{5}{\overset{3}{\times}} 6$$

$$\frac{2\ 7\ 3}{1\ 3\ 1}$$
$$\frac{2\ 7\ 3}{8\ 1\ 9}$$
$$\frac{2\ 7\ 3}{3\ 5\ 7\ 6\ 3}$$

In both cases, we add the digits of 273, and get 3 for the top of the cross, add the digits of 131, and get 5, for the bottom of the cross, cross multiply to get 15, which added gives 6 for the right hand side of the cross. Then in both cases we add the digits of 35763, giving 6, which appears on the left hand side of the cross, and the individual multiplication and division both prove, though one cross will suffice to show both operations.

Now let us go back to the original problem, with the remainder left in. It is obvious, that when we perform the final step in proving the original division, that is, adding the digits of the number divided, 35786, to obtain the digit 2 that appears on the left hand side of the cross; this number, 35786 is larger by 23, than the number 35763, we used when the division came out even.

Therefore we must add 23 to the series number 6, we obtained in summing up the digits of the smaller number, to get the same digit

that was obtained for the larger number, which includes the remainder. $6+23=29$, $2+9=11$, $1+1=2$, which is the digit that appears in the left hand side of the cross in our original division, when the remainder is still in.

Since in actual practice, we always add in the remainder, when we sum up the digits of the number divided, we must also add it to the other side of the cross to keep the two sides of the cross equal, equals added to equals remain equal. It is for this reason we are instructed to add in the remainder, after the cross multiplication has been performed.

It should be obvious from the above discussion, that the same basic principles that operate in multiplication, also apply to division, and therefore the same basic limitations hold for both.

It would seem that almost any one who uses multiplication of division to any extent could profit by becoming familiar with this simple method of checking. A clerk working in a hardware store is making an inventory, and takes pride in turning in an accurate set of figures. 17 saws @ $4.59, 35 chisels @ 76¢ etc.; the clerk, with almost no extra effort can check as he goes along.

A student is writing an exam in physics problems. It is most disconcerting, afterward, to find one has used the right method, but have your grade reduced, because of an error in multiplication.

The writer has worked problems, in which numerous multiplications and divisions occur in the same problem, with additions or other operations in between. Under such conditions one should carefully recheck each operation, as you proceed, or two workers should arrive at identical results independently. But who follows this rule, when you are alone, in a hurry, and anxious to know what the result is? It is most frustrating however, on checking later in such cases, to find an error has been made in the early part of the work, making all the original calculations that follow the error, useless. It should give a nice comfortable feeling, that the digits on both sides of the cross are checking, when working under these conditions.

When I first learned of this method I felt that the French make a mistake in teaching this at the grade level. My opinion was, that this method should be withheld till the high school level was reached, when both the limitations, and the reason the method works, could be understood. I thought a good place to introduce this, is when the study of "Series" is reached in algebra. It is rather difficult to find a practical example, to show that "series" are useful, at the high school level, useful as "series" become in higher mathematics. Here is an example that can be readily understood, even though the application happens to be a rather unusual one.

But when I taught the method to my own children at the grade

level, my attitude changed completely. In the first place, I found that even young children readily grasp the theory of compensating errors, though of course the reason the method works is beyond them. This can be taught later.

This method, when taught at the grade school level, may have more value as a teaching method, than as the check for which it was intended, in that it develops accuracy, and is a wonderful tool to eliminate faulty habits. This is because the child's attention is called to any error made, *immediately after* the error is made. Though faulty habits may occur in an almost infinite variety of forms, let us develop a hypothetical case. A child comes back to school after summer vacation, and is somewhat hazy on the multiplication and addition combinations. Such a child, for example, may get into the habit of occasionally saying $9 \times 7 = 64$. This child, if you pinned him down would know better, and does not make this mistake all the time, but does it frequently enough, so that when he says this to himself, in making a multiplication, it does not sound wrong to him, as it obviously would if he said to himself $9 + 7 = 35$. Such a faulty habit becomes almost impossible for a teacher to single out, in an individual when it occurs in a class of 20–30, or even more, since it is not consistently done.

But if the teacher stresses that the child should always try to find the nature of his own error before trying to correct it, when an error shows up by the proof by nine, the child will eliminate his own faulty habits. Did I say $9 \times 7 = 64$, did I make a mistake in addition, carrying, etc. etc.? The child will soon say to himself, "here I have said $9 \times 7 = 64$ again, I've got to watch that in the future." The blue pencil of the teacher on corrected papers, theoretically should have the same effect, but by that time the error is no longer fresh, and what normal child is going to go over a paper full of blue pencil marks. Unless forced to review it, it is consigned to the waste-basket. No wonder the French regard the method so highly.

It should be noted, that, regarded as a method of teaching, the fact that the method can occasionally fail because of the development of a compensating error, becomes relatively unimportant. If the child corrects himself 999 times out of 1000, and fails to correct himself the 1000th time, this is a matter of no great consequence.

Because of its value as a teaching method, I now feel that superintendents and principals, would perform a real service to the children, for whom they are responsible, if they would call attention to this article, to the grade teachers under their jurisdiction, as well as to those who teach at the high school level.

A UNIT OF WORK ON SOUND*

AN ENRICHMENT PROGRAM FOR PRIMARY CHILDREN

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A phase of physical science was stressed in the first grade of the 1954 Summer Laboratory School at the University of Wisconsin. Sound was our center of interest. Special teachers of art, music, German, physical education, speech and library contributed much to this program through providing enrichment experiences.

Sound was the area chosen by the teacher for these reasons:

(1) The learning experiences involved offer many and varied opportunities for meeting the common needs of primary children.

(2) All communities have resources related to sound which the children can utilize in their learning experiences. This helps them gain functional knowledge since direct experience and knowledge go hand in hand. Understanding their environment helps children to live with satisfaction to themselves and to others.

(3) The daily experiences involved in developing the desired understandings, skills, and attitudes can stimulate the child to observe and question. These experiences serve to challenge the child to think for himself and to think with others.

(4) Most of the equipment needed is simple, inexpensive and easily cared for. Much of the equipment can be brought from home and can be used in other learning situations. Technical scientific equipment is not necessary; however, some simple equipment, such as a tuning fork, will add to the effectiveness of learning and to the level of interest of this age group.

(5) The learning experiences can motivate the child to engage in an activity during his free time. This is a time when a child may be face-to-face with others in a small group situation or by himself. It is a time when a child is free to make his own choices; whether it be to just talk, to think, to experiment, to investigate, or to show others his findings. Free time in school provides opportunities for the interested teacher to observe and question the child's understanding of his independence and responsibility.

Experimenting, observing, discussing, reading, writing, hearing phonograph records (music and story), making murals, dramatizing, composing stories, keeping records and making choices all are activ-

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ities which promote growth in independence and responsibility. Independence and responsibility work together. The responsibility that the child assumes has much to do with his acquiring knowledges, and developing attitudes and skills essential to democratic living.

CONTENTS

Purpose, Understandings, and Possible Approaches
Some Suggested Understandings and Activities
Charts (developed during discussion periods)
Picture Stories ("Air de Ballet" was played, children drew action pictures, later composed stories)
Evaluation cards (children read the cards, followed directions and discussed findings)
Stories (three groups were formed, each group chose a topic, sound events were suggested by each child, sequence of events were planned, then story was composed)
Bibliography

Purpose:

To provide experiences for primary children so that they will have opportunities:

to grow in their ability in solving problems through the use of facts;
to interpret what they see about them and appreciate their environment;
to grow in developing a scientific attitude of mind:
to widen their range of interest.

Understandings: (Pella's list)

There are many different kinds of sounds.
We can identify some things by the sounds they make.
Some sounds are pleasant and some are not.
Sounds that are unpleasant are called noise.
Sounds are caused by vibrating objects.
Sounds travel out in all directions from source.
Sound travels around corners.
The farther away from a sound you are, the fainter it gets, and the closer you are, the louder it is.
Sounds help us.
Sound affects the ears.

Other Suggested Understandings:

Some things vibrate so slowly that we cannot hear them.
We hear things because the vibrating object sets the air around it vibrating.

The loudness of a sound depends upon how hard the object is vibrating.

Air carries sound to our ears.

Sounds may be produced in many ways.

Materials other than air carry sound—wood, water, strings.

Possible Approaches:

Discussion of sounds heard in the room.

Rhythms—showing how different music makes us feel.

Reading a book with sounds, such as *Boku and the Sound* or *Quiet Mother and the Noisy Little Boy.*

SOUND

Some suggested understandings and activities for primary children

1. There are many different kinds of sounds.

   a. Make sounds in the classroom and listen to the differences.

clap hands	snap on lights
sing	rustle paper
talk	whistle
hum	scratch
shut door	squeak chair
turn on faucet	tramp of feet
snap fingers	write

   b. Sit quietly with eyes closed. Make a class chart of sounds heard.

honk	bus
train whistle	people walking
wind blowing	dog barking
airplane	child crying
truck	air pump

   c. Make groupings of sounds (individual or committee projects).

weather sounds	in the park
on the farm	on the street
in the stores	at home
on the playground	on the river
at the railway station	at the circus

2. We can identify some things by the sounds they make.

   a. Collect pictures of things outside of the classroom that can be identified by sounds (committee work).

   hail, rain, wind
   animals
   musical instruments
   trains, airplanes, cars
   electric fan, refrigerator
   dishes
   phone, vacuum cleaner

   b. Listen to record, "Sounds Around the House," Part I, and identify the sounds.

sawing	barking
hammering	sweeping
water (faucet)	egg beater
painting	flour sifter
vacuum cleaner	door
door bell	etc.

c. Listen to record, "Sounds Around the House," Parts II, and draw the sounds. Use same color for all sound pictures or choose different colors as sounds are drawn.

)))..   stirring and tapping with spoon

O egg beater

X bottles clink

/ water faucet

vacuum cleaner

d. Make sounds with various materials. Make sounds behind a screen—others identify the sound and its source.

clock—ticking, alarm
piano—play a high note
—play a low note
snap on lights
drop a chain slowly in a pan
clink two glasses

snap snapper
crush a lump of sugar
slap a piece of wood on table
sounds with rhythm intruments
pour water
cough, sneeze

3. Some sounds are pleasant and some are not. Sounds that are unpleasant are called noise

a. Talk over how sounds make us feel.

gay   sad   in a hurry
happy   weepy   good
sleepy   hungry   like working

b. Play a variety of music—note restfulness and stimulation—(piano or records). Children interpret the music freely.

running   jumping   bouncing
hopping   swaying   balls
skipping   "waltzing"   sawing
    hammering

c. Discuss which sounds are pleasant.

some music   friends playing
family sounds   company coming
some voices   zoos, circuses
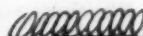(animals)   room sounds

d. Discuss which sounds are unpleasant.

angry voices   air pumps
children playing   breaking bottles
squeaking brakes   scratched record
some room sounds

e. Listen to voices and discuss "A pleasing voice."

talks so we can hear
just "says" words (speaks distinctly)
a "story" voice (a voice with feeling)

f. Discuss why we turn off our radio when there is much static.

not good for the radio
too much noise, can't hear what we want to hear

4. Sounds are caused by vibrating objects.

a. Place hand on throat when speaking or singing and discuss the feeling.

feel the vibration

b. Construct a "xylophone." Use seven tumblers (alike) with varying amounts of water. Tap each glass gently with different objects, choose the kind of object that helps make the best sound.   different tones will be produced

c. Place paper clips on drum. Strike down on drum with hand.   Clips will bounce indicating vibration

d. Stretch rubber band and pluck it.   Rubber band will vibrate

e. Strike a tuning fork on hard rubber.   Hear the sound
Hold fork to ear   Feel the sound
Stop vibration with hand   See the sound
Touch fork to nose
Touch corner of card gently
Hold vibrating prongs in water

f. Drop a drop of water in a pan of water.   Notice the vibration circles

g. Make a chart of animal sounds which are made by various parts of the body.   throat   feet   rattles
wings   tail

5. Sounds travel out in all directions from source.

a. Play sound games.

Find It

All children have the same kind of noise maker on their desks. Three children scatter away from the group in different directions and close eyes. One child in the group makes the noise.   Hear the sound
Identify the direction of the sound
Name the child

6. Sound travels around corners.

a. Children talk to each other around corners.   Sound travels around corners

b. Discuss what is said and why it could be heard.

7. The farther away from a sound you are, the fainter it gets, and the closer you are, the louder it is.

a. Listen to moving away things.   trains   clocks

b. Listen to things getting closer.   trucks   rhythm instruments

8. Sounds help us.

    a. Discuss ways that sounds help   to enjoy things
       us.   to find our way
                                   to find someone who is lost
                                   to tell when there is danger
                                   to order things from the store
                                   to call a doctor
                                   to tell news

9. Sound affects the ears.

    a. Discuss care of ears.   how to talk to people
                                   how to keep ears clean (only a doctor
                                     should really clean ears)

    b. Talk about reasons why good   to be friendly with people
       hearing is important.   to learn in school
                                   to enjoy music and other things

SUMMER SCHOOL WISHES

We would like to do many things in summer school.
We would like to do these things.

Read		Paint	
	Play		Talk
Draw		Sing	
	Talk Another Way		
Hear Stories		Spell	
	Experiment		
Arithmetic		Make Things	
	Do Other Things		

FINDING SOUNDS

We can find different sounds.
We can find different sounds at different places.
Sometimes we can find the same sound at different places.
Here are some sound places:

our homes	our schools
the farms	our churches
the stores	the airport
the parks	the circus
the streets	the zoo
the playgrounds	

HEARING SOUNDS

We can hear many sounds in our room.
Here are the sounds we have heard:

honk	buzz
squeak	gasp

roar	rat-a-tat-tat
bow-wow	r-r-r-r
swish	rustle
click	

The following sound words were found in library books and were added to the list by the children:

clip-clip	tap, tap
bang	pop, pop, pop
toot toot	whoot
mew mew	smack
caw caw	crunch
cluck, cluck	gobble
puff	gulp
splash	ker-chug
cock-a-doodle-do	Wrrrrrrrr
quack	dong! dong! dong! dong!
wuff	

MAKING SOUNDS

We can hear many different kinds of sounds when we do these things:

talk	write
color	telephone
walk	breathe
sing	
open the desk	play the piano
shut the desk	pull down shade
sharpen pencils	wash out bottles
move the chairs	open windows
snap on lights	shut doors
turn on the faucet	

THINGS MOVE, SOUNDS COME

Sounds seem very strange.

Sometimes we hear them everywhere. We hear them outdoors and we hear them indoors.

Sometimes we do not hear them at all. We listen and listen. We do not hear a sound. Everything is very still.

Ding-dong, ding-dong!

Tweet, tweet, tweet!

Honk! Honk! Honk!

Things are moving. Sounds are coming.

EVALUATION CARDS

Do, Think and Tell	Choose, Listen, Think and Tell
Place the clock in the center of the table. Listen carefully as you walk around the table quietly. What can you tell?	Talk with three friends. Listen carefully to their voices. Did you find some pleasing voices?
Can you tell?	**Think and Tell**
Why do you turn off your radio when there is much static?	What sounds can be heard far away?
Think and Tell	**Do and Tell**
What sounds do you like to hear? What sounds do you not like to hear?	Hold a clock near your ear. Move it away slowly. What can you tell?
Do and Tell	**Think and Tell**
Place your hand on your throat. Talk to yourself or a friend. What can you tell?	Think about animal sounds. What do the animals use to make sounds? (If you need help, look at the bulletin board in the back of our room.)

PICTURE STORIES

Ballet dancing on the stage.
Tap, tap goes their shoes.
> Joanne K.

Peter is going to eat dinner.
Peter goes crunch, crunch, crunch.
> Denise

We are marching and marching.
Tramp! Tramp! Tramp! Tramp!
> Stephen C.

The ball bounces and bounces.
Up in the air it goes.
When it comes down, it makes a noise.
Bounce! Bounce! Bounce!
> Roberta

Up and down goes the handle.
Out comes the water.
Splash goes the water as it hits the pail.
Splash, splash, splash.
> Steven A.

Bow-wow goes the dog.
Bounce, bounce goes the ball.
> Junji

Bouncing! Bouncing! Bouncing!
Funny bouncing balls.
Bouncing balls are fun to bounce.
> Margaret

Henrik and Junji dance like the music goes.

Tap, tap, goes their feet.
Then Miss Rowe says "stop" and she stops the music.
> Henrik

This is Nancy bouncing a ball.
Bounce, bounce, blunce.
> Linda

See the cows.
Moo-moo-moo-moo.
They are ready to be milked.
They want their hay and oats.
> Paul

Bow-wow, bow-wow-wow.
That's my dog Squeakie.
My brother and I are playing catch.
My dog caught the ball three times with his mouth.
> Tommy R.

A man is sawing a piece of wood.
He is going to build a house for himself.
The saw makes this sound.
Z-z-z-z-z-z-z-.
The hammer makes this sound bang, bang.

Joel

The girl in my picture is **Mary**.
She is bouncing a ball.
Bounce, bounce it goes.

Rosemary

This is a man cutting wood.
He is also cutting the bark off.
Z-z-z-z-z-z-z goes the saw

Cheryl

I pound nails with my hammer.
Pound! Pound! Pound!

Mary

THE CIRCUS TRIP

Betsy and Bill were laughing, talking and skipping as they went down the street with Father. They were going to the circus and going to the circus with Father was fun.

Many people were going to the circus. Tom Jones was whistling as he rode past on his bicycle. Sally Smith and her father honked their horn when they went past. Some people were walking and some people were riding.

Betsy, Bill and Father needed to stop. Other people stopped too. They had come to the railroad crossing. They could hear a train coming and the signal bell was ringing. Soon the train went rumbling by and the signal bell stopped ringing. Then everyone hurried to the circus.

Buying tickets was fun. Betsy gave the ticket man five dimes and a quarter. Bill gave him three quarters. But Father gave the ticket man three half-dollars.

Bill knew they were going to see lions because he could hear the lions roaring.

Father knew they would need to hurry because he could hear a man telling them to hurry. The circus was ready to start. The band was playing.

They got in their seats just as the funny little clown with the big flip-flop shoes went past. He was doing many good tricks. Everyone laughed at the funny little clown.

There were many interesting acts. Everyone watched closely. Sometimes everyone would scream and yell. Sometimes they would ah-h-h and oh. And sometimes every one was very quiet. Being at the circus was so much fun.

When they were leaving the circus they could hear the popcorn man calling. Father bought some popcorn for them and they ate popcorn all the way home.

Mother was waiting to hear about their good time. Bill told about

the tigers, seals and elephants. Betsy told about the trapeze artists, the beautiful white horses and the dancing dogs.

Mother knew she would hear more about the circus tomorrow. And she knew Betsy and Bill would see circus acts in their dreams.

And Mother was right.

SOUNDS EVERYWHERE

We hear sounds everywhere.

Many sounds make us feel happy. Some music makes us feel like dancing gay dances. Some music tells us to dance like big strong people and sometimes it tells us to dance like little people.

We can run, walk, skip, tiptoe, jump, sway back and forth and many other things when the music tells us to.

We like to sit and listen to music too. We like to hear people sing and the orchestras play.

Hearing our friends laughing and talking makes us feel good. We can tell they are having fun.

We like to hear the sounds in the woods. The squirrels chatter and chatter. The birds sing their songs. The rabbits go hopping here and there. And sometimes the wind makes the leaves rustle.

Listening to the sounds in the stream is fun. The ducks splash and quack. The frogs croak. The beaver slaps his tail on the water. And the water ripples and ripples.

We think family sounds are the best sounds.

Some sounds make us feel sad. We feel sad when we hear a baby cry and cry, and when we hear someone is lost.

Family sounds are the best sounds.

SALLY AND JACK ON THE FARM

Sally and Jack helped Grandmother wash the breakfast dishes every morning. Sally liked to rinse the milk glasses under the faucet. Jack liked to dry the dishes and put them away. And Grandmother told interesting stories about the farm as she washed the dishes.

One morning as they were working, Jack heard a very faint sound. He listened carefully. He knew it was the sound of an airplane and he wondered if it were coming near Grandfather's farm. Yes, it was! The sound was getting louder and louder.

Sally heard the sound too.

Jack and Sally hurried outdoors. They hurried so fast that they slammed the screen door. They ran down the wooden porch steps and across the yard to the garden fence. This was their favorite watching place.

When Jack and Sally got to the fence, the airplane was high above them. The engine roar was very loud and the airplane looked very big.

The children watched and listened. They watched and listened until the engine roar was very faint and the airplane looked very tiny.

All at once Sally felt a scratch, scratch on her shoe and heard a tiny purr. She looked down and saw a little black kitten scratching and purring.

Jack picked up the little kitten and carried it back to the barn to Mother Cat. Sally skipped along as she listened to the kitten purr and purr.

Grandfather was in the barn too. He was busy giving oats and hay to Brownie. Grandfather said they could ride Brownie when he stopped eating.

So Sally watched Brownie eat and Jack helped Grandfather gather tools. Grandfather needed to fix the pasture fence and check the cows and calves.

Soon they were on their way to the pasture. Sally was having a good ride. Brownie was walking slowly and Grandfather and Jack were walking beside them.

Jack had a good ride coming back to the barn. Brownie walked very fast. Grandfather and Sally walked very fast too.

When they got back to the barn, there was trouble in the pig pen. One little pig was squealing and squealing. He was caught in a board fence and was squealing for help.

Grandfather knew how to help the little pig. Soon the little pig was squealing happy squeals.

Sally and Jack took Brownie into the barn and hurried back to watch Grandfather fix the board fence.

When Grandfather stopped pounding, they could hear the dinner bell ringing. Grandmother was calling them to dinner.

After dinner Sally and Jack took a nap. They were very tired.

Jack went to sleep first.

Just as Sally was going to sleep she heard a soft pitter-pat on the window pane. It was beginning to rain.

Sally smiled a little smile. She knew it was a good time to take a nap.

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ILLINOIS STATE NORMAL UNIVERSITY ANNOUNCES THE EIGHTH ANNUAL CONFERENCE ON THE TEACHING OF MATHEMATICS—ELEMENTARY AND SECONDARY LEVELS

MARCH 26, 1955

General theme: Individual Differences

Mark the date on your calendar—March 26, 1955, from 9:00 A.M. to 3:00 P.M., with a social hour following the conclusion of the afternoon session.

The place—On the campus of Illinois State Normal University.

It has been our good fortune again to secure outstanding speakers for the conference.

   Dr. William David Reeve, professor emeritus, Columbia University, will address the secondary teachers. The title of his talk is: "The Problem of Varying Abilities in Mathematics."

   Dr. Charlotte Junge, professor of education, Wayne University, will address the elementary teachers.

Following these principal addresses, there will be group discussions of interest to teachers of every level: beginners, intermediate, upper grades, and high school. Each of these discussions will follow the general theme of recognizing, utilizing, and providing for the varying abilities in mathematics.

We anticipate an outstanding conference, and extend to you a personal invitation to attend its sessions. Bring with you any one who is in any way interested in the teaching of mathematics.

<div style="text-align: right">

T. E. RINE, *Chairman*
1955 Mathematics Conference

</div>

THE ROLE OF PHYSICS IN THE EMERGING
HIGH SCHOOL CURRICULUM*

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INTRODUCTION

Within the last few years a number of facts have been discovered about high-school physics that should cause the teacher of high-school physics a great deal of concern. Most of these facts appear in a report[1] of a committee of the CASMT. In essence the report indicates that in terms of percentages fewer than one-fifth as many students elect physics today as in 1890. Within the past three or four years the decline has been greater than every before. Further, fewer than half the high schools in the United States ever offer physics, and of those that do, many offer it on alternate years with chemistry. Another report[2] dealing with the training of science teachers is even more depressing. In several states with which the report was concerned, about half the teachers of physics had no background in physics other than the introductory course or an incidental survey course in physical science. Also, in one populous state in 1953 not one student from any of the teachers colleges or universities who graduated with a major or minor in physics entered the teaching profession.

It was perhaps a knowledge of many of these facts that led Hurd[3] to state that . . . "[physics] does not fit into either the high school or the college pattern of modern education." To have such a viewpoint expressed by a science educator from a large university is indeed distressing. As a personal opinion, this writer would like to state emphatically that he disagrees with Hurd, that the logic used by Hurd is suspect to invalidity, and the citations used are not necessarily blessed with authority. Yet the article does merit a great deal of study, perhaps more than it has received since the Editor of SCHOOL SCIENCE AND MATHEMATICS *has not received one letter* protesting Hurd's interment of physics.

In view of all these negativisms about physics, it would seem rea-

sonable that the course might be allowed to pass more or less grace-fully out of existence. Yet there are a number of facts of overwhelm-ing significance that point to the unequivocal need for physics in one form or another throughout the public school. Further such facts point to a need that is becoming steadily greater every day. In fact, this writer would postulate that the enrollments have in many cases declined because many organizations of physicists and physics teachers have failed to recognize the tremendous implications of the physical sciences throughout the entire curriculum.

An examination of a number of research studies tends to negate further any belief that physics does not fit into the pattern of high-school or college education. In one of the monumental studies of science education, Wise[4] attempted to develop a list of principles in the physical sciences that were of importance for the *general* (not special) education of students in secondary schools. Of the two hun-dred seventy major principles thus identified, two-thirds were from the field of physics, about one-quarter from chemistry, and the re-mainder from geology.

An examination of the popular textbooks of general science will show that about sixty per cent of the content comes from the physical sciences, and about two-thirds of that from the area of physics. Studies too numerous to cite here that deal with the general as well as the scientific interests of adolescents, indicate that most adolescents have some interest in science, and many have interests in science that are much more intense than interests in other areas. Of those who profess major interests in the area of science, activities involving the field of physics tend to predominate.

As a final point one might well indicate that the areas in science in which greatest progress have been made in the last few years (rockets, jets, television, electronics, atomic fission, thermonuclear fusion), and which demand of the layman more and more attention, require a knowledge of physics. Hence without physics, the appalling scien-tific illiteracy of both the lay and scientific population will grow.

In other words, "Hurd buried the wrong body."

WHERE STANDS PHYSICS?

The preceding discussion of course illustrates an anomaly. If physics is so important, why it it dying out? Obviously, in order to suggest an answer, one must commit himself to one or more premises. Here they are:

1. Physics in one form or another should be included in the curric-ulum of the public school at every grade level.

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[4] Wise, Harold E., *The Major Principles of Physics, Chemistry and Geology of Importance for General Educa-tion.* Circular No. 308-IV, Selected Science Services, Division of Secondary Education, Office of Education, Federal Security Agency. Washington 25, D. C., 1948, Pp. 1-18.

2. The physicist and teacher of physics must realize that the study of materials of physics is of value for many persons other than the genius, the college bound, and the future physicist or physics teacher.

Once these premises are accepted a program must be undertaken to implement them. What are these methods of implementation that place physics in its proper perspective?

At the Elementary and Junior High School Levels

The subject-matter of physics found in elementary and general science consists of the qualitative and descriptive study of areas such as heat, light, electricity, the atom, fuels, sound, radio and television. To state that the usual courses in introductory physics that are designed for students intending to major or minor in physics are suitable for teachers of elementary or general science is indeed ridiculous. The teacher of elementary or general science needs this mathematical and quantitative physics "like she needs a hole in the head." She needs courses built around the qualitative and descriptive study of areas ordinarily taught at these levels. Yet few if any colleges provide such courses, although most provide survey courses in physical sciences that are treated with contempt by the college physics teacher. Until physics departments devote time and effort to developing courses in physical sciences that are worthy academic offerings for teachers and give their wholehearted support to the purposes for which they are designed, teachers of elementary and general science will continue to avoid both the introductory courses in physics (as not being meaningful) and the survey courses in physical science (as being cheap and watered down). It is indeed a sad commentary on college physicists if they cannot or will not recognize that there is more than one kind of course in physics. If such recognition is not forthcoming the physics content of courses in elementary and general science will become less and will be taught more poorly. Such experiences with physics have probably helped cause the declining enrollments in high-school physics in the past and will continue to do so in the future.

Enlightened self interest should encourage departments of physics in colleges "to change their ways of living" with respect to this problem.

In the First Two Years of High School

In the past few years several articles[5,6] have appeared in which the values of a general education course in physical science for *all* high

⁵ Mallinson, George Greisen and Buck, Jacqueline V., "The Case for General Physical Science." *Metropolitan Detroit Science Review*, XIII (May, 1953), 24–26.

⁶ Mallinson, George Greisen and Buck, Jacqueline V., "The Coming of General Physical Science." *The Clearing House*, XXVIII (November, 1953), 158–161.

school students are discussed. Hurd, in his article already cited, also recommends the development of such a course for the secondary school. However, his support is not likely to evoke much enthusiasm from those who have worked most with the course. Specifically, he makes two points:

1. ". . . physics is slowly being eliminated from the high school curriculum and a type of science course [general physical science] more in line with modern educational points of view is taking its place."[7]

2. ". . . the majority of science principles [for this course] were drawn from physics, many from chemistry, plus a smaller number from geology, astronomy, and meteorology."[8]

One might well infer from the first point that this course is a substitute for or designed to replace physics. It is such a viewpoint that has been responsible for the antipathy against it. General physical science is designed to stand on its own feet as a general education course in physical science for all students. It is designed to serve as a terminal course for those who do not desire to taken the more specialized courses in physics and/or chemistry in the junior or senior years of high school, and to serve as a prerequisite for physics and chemistry for those who do desire to take them. The students who do take chemistry and physics will thus be better prepared. Where such a plan for general physical science has been followed, enrollments in physics *have increased*, which is heartily to be commended.

From the second point one might infer that in developing such a course the various fields of physical science are first investigated. Then principles in their proper proportions are taken from them and are then integrated into a pattern around which learning experiences are developed. This is inconsistent with the thinking of most persons who've worked with the course. In general, the major problems of physical science that face students and citizens are identified, the principles of physical science (not physics, chemistry, etc.) that ramify into these problems are selected, and these are developed into learning areas around which classroom experiences are built. No efforts should be made to select topics, principles, or other fragments that are to be fused into general physical science. Such efforts have been tried many times in the past and have produced courses that died "a much needed death."

Those persons who are interested in the future of physics must work earnestly to develop the course in general physical science in light of the philosophy just mentioned. If the attitude prevails that

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[7] *Op cit.*, p. 447.
[8] *Ibid.*

it is designed for those not sufficiently competent to study physics it will not serve its purpose as the general education course that is a prerequisite to physics. Physics will then be a new subject to high-school juniors and seniors and they are likely to shy away from it. Witness enrollments in physics during the last few years!

At the Junior and Senior Years of High School

At this point the writer would like to suggest that the measures of implementation already described will induce more persons to take physics. However these measures are not likely to be enough. The writer agrees with Hurd that few courses at the high-school level have remained so traditional as physics. Despite the many discoveries in the physical sciences, courses have failed to take advantage of them as possibilities for vitalizing the learning experiences in physics. The course trudges on with experiments that are deductive-illustrative in style rather than inductive, activities that seem to be essentially designed for disciplining the mind, and workbooks that have blanks to fill. In other words, the emphasis seems to be on memory rather than *thinking physically*.

The writer suggest that the traditional areas of study, namely, mechanics, sound, heat, light and electricity need to be overhauled. Efforts need to be made to build a course in physics around problem areas of science in which physics applies. Such areas may include energy under control, energy out of control, the atom as a source of energy, electronics, radiation, energy in communication, and energy in transportation. These of course are merely suggestive since the writer has not investigated the matter thoroughly. But, they may serve as stimuli to those to whom physics is a major interest.

However, the author wishes to emphasize that such a course should be rigorous as well as practical. The student should emerge knowing more physics and how better to apply it because of his earlier experiences with physics and because of the changes that are educationally desirable.

CONCLUSIONS

Obviously, this report covers only a few of the points (the writer hopes they are key ones) with respect to the place of physics in the emerging curriculum. The emerging curriculum does have a place for physics—a bigger one than ever before. However, it does recognize the general education value of the qualitative-descriptive study of physics in the earlier school years. Such activities do not ultimately cheapen physics any more than basic arithmetic cheapens trigonometry, or basic English skills cheapen the study of literature.

All these cases are illustrative of building up to the rigorous study

of physics rather than having the student jump from the nadir of physics to the mountain top in one leap.

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THE EVALUATION PROGRAM IN SECONDARY MATHEMATICS

WILLIAM DAVID REEVE

Professor Emeritus of Mathematics, Teachers College, Columbia University
(Concluded from February)

4. CRITERIA FOR CONSTRUCTING NEW-TYPE TESTS

Guiding principles underlying good tests. The following guiding principles underlie the construction of a good test in mathematics:

1. *A test should attempt to increase the student's ability to master the subject matter that has been presented to him.* This means that the teacher must discern clearly the objectives in the topic or course and must build the test so as to measure the extent to which these objectives have been realized. Such a procedure measures the progress made by a student or a class. This is frequently taken as the first aim of an examination. A test which has no reference to what has already been taught cannot meet this requirement. If students who have the native capacity to learn a certain thing fail to do so, there is an obvious opportunity for remedial work. On the other hand, there is little to be expected from or gained by remedial work in the case of students who have done as well as can be expected considering their innate capacity. If both the diagnosis and the subsequent remedial work suggested by the tests are to be of value, the teacher must have at hand specific measurements of the student's achievement. Such measurements can be fairly made only in a field in which the student has been working.

Moreover, tests that contain material that has not been previously taught cannot have much diagnostic value. In such a case it is impossible to tell whether poor results mean that the content of the course was too difficult, that the teaching was not well done, or that the material was entirely unknown to the students.

2. *Every test should emphasize those parts of the subject matter which are fundamental and to which the students have directed the most attention.* Nothing should receive attention that is not worth perpetuating in the course. This means that every test should contain a thorough sampling of the fundamental ideas of a topic or a course for the complete mastery of which the student is held responsible; that is, the test must be *comprehensive*. This has never been true of the traditional "essay-type" examination.

If the two preceding principles are adopted the test will be ranked as *valid* or as having *validity*—that property of a test which describes *the degree to which a test measures what it is intended to measure.*

3. *The scoring of every test should be made as objective as possible.* In this way different teachers employing the same test in measuring the same abilities may obtain exactly the same results. Moreover, the same teacher, in marking a test a second time, should obtain the same results that he got the first time. This means that the personal factor must be largely removed from the scoring of results and that teachers in framing the tests should be careful about the mechanics of them. If this principle is kept in mind, not only will the response of the students be more uniform, but the marking practices of those who score the papers will be less variable. These ends are achieved by making the number of items in the test as large as necessary and by requiring for each of these items a definite response to which all persons scoring the test papers will readily agree. The failure to do this is responsible for the chief defect of many "essay-type" examinations.

4. *Every test should be reliable. Reliability* is that property of a test which determines the degree to which repetition of the test will give the same results. A test may measure what it is intended to measure, but it may do it very unreliably. The determination of the reliability of a test is, however, a more technical matter and need not be further discussed here. It is discussed fully in books on testing.

5. *Every test should be so constructed that it is almost self-administering.* It should be made so that it can be easily given and scored by an intelligent person who may or may not have had much mathematical training. In the past this has not often been done. The motto should be: "Hard to make but easy to give and score."

6. *Every test should make it possible to set some sort of standard of achievement for a student.* Such a standard may match him against his own group or against his own record. The attempt should therefore be to devise tests that measure adequately the student's knowledge of the subject matter studied by the majority of the class, to assist the teacher in selecting only the important topics, to stimulate the student to greater effort, and to aid him in self-instruction.

The testing of specific objectives. The method of procedure in making each test can best be understood by taking certain definite objectives, upon which it will be assumed we are agreed, and building up the practice tests and composite tests which seem to be necessary to ascertain to what extent our aims are being realized. Let us assume for the sake of illustration that we have decided upon the following specific objectives in teaching the formula:

1. *To develop certain rules of procedure and to translate them into formulas.* This means that students should understand the meaning of formulas as shorthand rules of mathematics. These rules should, in general, grow out of their experience. This is where algebra properly begins.

2. *To translate certain formulas into rules of procedure.* This means that students must know how to use a formula when the need arises. Obviously, they cannot do so unless they can translate the formula into a rule of procedure.

3. *To evaluate certain formulas.* This means that we are to find the values of certain letters when the values of the others are known. The formulas should be of a difficulty no greater than that found in the operations which the students have been taught or which they may be expected to understand.

Four types of equations involved are as follows:

a. $2w = 16$ c. $p + 4 = 104$

b. $\frac{1}{2}h = 4$ d. $n - 4 = 7$

The need for solving an equation like $2w = 16$ or $16 = 2w$ arises in using a formula like $A = lw$. For example, if the area of a rectangle is 6 and the length is 2, the width is found by solving the equation $6 = 2w$. The necessity for solving the second type arises in using a formula like $A = \frac{1}{2}bh$. For example, if $A = 4$, $b = 1$, and the height is to be determined, we must solve the equation $\frac{1}{2}h = 4$, or $4 = \frac{1}{2}h$. In like manner, if in using the formula $A = p + i$ we know that $A = 104$ and $i = 4$, and need to find p, we must solve the equation $104 = p + 4$ or $p + 4 = 104$. Similarly, if in the formula $A - p = i$ we know that $p = 100$ and $i = 5$, and wish to find A, we must solve the equation

$$A - 100 = 5, \quad \text{or} \quad 5 = A - 100.$$

4. *To derive one formula from another.* This means that the student must be able to solve a formula for one letter in terms of the other letters involved. Some writers refer to this as "changing the subject of the formula," but this phrase is not a good one to use in the classroom because it is likely to be confusing to the student and thus adds to his difficulty.

If the students have sufficient drill in solving the first four types of equations given above, they will be able to derive one formula from another. Thus, if they can solve such equations as $2w = 16$ for w, they will be able to solve the equation $lw = A$ for w, and so on. Care should be taken, however, to develop the work inductively.

5. *To represent certain formulas by graphs.* This involves the ability to make a table of values for a formula. The limit of difficulty in this

work should be the Fahrenheit-centigrade formula $F = 9/5c + 32$ or $F = 1.8c + 32$.

6. *To understand the idea of dependence of one quantity upon another.*

The tests that follow on pages 219–227 are samples of those which might be used in connection with the list of objectives given above.

It is not expected that these samples will cover every need. The rules of procedure to be translated into their formulas will vary widely according to local needs. Most of the rules of procedure given below, however, will be included in every list that is taught.

TRANSLATING RULES OF PROCEDURE INTO FORMULAS

Translate each of the following rules of procedure into its correct formula:

1. The area of a triangle is one-half the product of its base and height.
2. The cost of a number of articles is equal to the number of articles multiplied by the cost of one article.
3. The sum of the three angles of a triangle ABC is $180°$.
4. The volume of a rectangular solid is equal to the product of its length, width, and height.
5. The volume of a pyramid is equal to one-third of the product of its base (area) and height.
6. The perimeter of a regular hexagon is equal to six times one side.
7. The distance covered by an automobile at a given rate in a certain period of time is equal to the product of the rate and the time.
8. The volume of a cone is equal to one-third of the product of its base (area) and height.
9. The area of a cricle is equal to the product of the constant *pi* and the square of its radius.
10. The area of a square is equal to the square of a side.
11. The perimeter of a rectangle is equal to twice the sum of its length and width.
12. The profit on an article is equal to the difference between the marked price and the cost.
13. The net cost of an article is equal to the marked price diminished by the discount.
14. The diameter of a circle is twice its radius.
15. The perimeter of a square is equal to four times a side.

The reverse test, the recognition of the meaning of formulas, or the translation of formulas into their rules, can readily be made from the test given above. All that is necessary to do is to write the formu-

las down on paper and ask the students to give the rules of procedure which correspond to them. Similarly, in the selection of the formulas to be evaluated it will be necessary to consider the local needs and the abilities of the students. The formulas selected need not be the same ones that were previously translated into rules, but they should be formulas which can be simply stated and which do not involve too complicated computations in their evaluation. A mastery of the essential parts of algebra depends to a large extent upon the power acquired in relatively simple operations, and for this reason emphasis upon reasonably rapid work in a large number of simple cases is desirable. In the following test both the formula and its description are given, and the student is required to match the two columns. In addition to the interest aroused by this type of test, the work is important in training the student to recognize at sight the important formulas of mathematics—an ability which will be extremely useful in subsequent work.

GEOMETRIC RULES AND THEIR FORMULAS

The right-hand column below contains the rules that are expessed by their corresponding formulas in the left-hand column, but not in the same order. Match the two columns by stating the letter which corresponds to the proper rule as shown in the first square:

Formula		Rule
1. $A = lw$	[e]	**a.** The sum of the angles of the triangle ABC is 180°.
2. $c = pn$	[ ]	**b.** The volume of a rectangular solid is the product of the length, width, and height.
3. $p = 4s$	[ ]	**c.** The perimeter of a rectangle is equal to twice the sum of the length and width.
4. $A = \pi r^2$	[ ]	**d.** The volume of a sphere is four-thirds of the product of pi and the cube of the radius.
5. $V = lwh$	[ ]	**e.** The area of a rectangle is the product of its length and its width.
6. $d = 2r$	[ ]	**f.** The distance covered by a moving body is the product of its rate (speed) and the time of travel.
7. $V = Bh/3$	[ ]	**g.** The volume of a cube is the cube of its edge.
8. $p = 2(l+w)$	[ ]	**h.** The surface area of a cube is six times the area of one face.
9. $V = e^3$	[ ]	**i.** The perimeter of a square is four times its side.
10. $c = 2\pi r$	[ ]	**j.** The volume of a pyramid is one-third of the product of its base and its height.
11. $A = \frac{1}{2}h(b+b')$	[ ]	**k.** The area of a triangle is one-half the product of its base and its height.

12. $A = \frac{1}{2}bh$ ☐   **l.** The cost of a number of articles at a fixed price is the product of their number and the price of each.

13. $S = 6e^2$ ☐   **m.** The circumference of a circle is the product of 2 *pi* and its radius.

14. $V = 4/3\pi r^3$ ☐   **n.** A lateral area of a cylinder is the product of 2 *pi*, its radius, and its height.

15. $V = \pi r^2 h$ ☐   **o.** The area of a circle is *pi* multiplied by the square of its radius.

16. $A = 2\pi rh$ ☐   **p.** The area of a trapezoid is the product of one-half its height and the sum of its bases.

17. $A + B + C = 180°$ ☐   **q.** The simple interest on a sum of money is the product of the principal, rate, and time.

18. $c = m - d$ ☐   **r.** The net cost of any article is equal to the marked price minus the discount.

19. $d = rt$ ☐   **s.** The volume of a cylinder is the product of *pi*, the square of the radius, and the height.

20. $i = prt$ ☐   **t.** The diameter of a circle is twice its radius.

In the practical applications of algebra the most frequent use that is made of the subject involves evaluation of some sort. Work of the kind given in the following test is also important in showing the students the wide range of cases to which a single formula can be applied and thus leading them to appreciate the great power of algebraic methods.

As has been previously stated, the help that the student can render in diagnosing his own difficulties should not be overlooked. In work like that given below, the student's errors will be largely due to carelessness, but the plan of having the student locate the source of his errors for himself is one that might well be followed more often than it is.

EVALUATION OF FORMULAS

Using the formula for the area of a rectangle ($A = lw$), solve the following exercises:

1. Find A when $l = 2\frac{3}{4}''$ and $w = 1\frac{1}{2}''$.
2. Find l when $A = 6.5$ sq. in. and $w = 1.3''$.
3. Find w when $A = 10$ sq. in. and $l = 8''$.
4. Find A when $l = 4.25''$ and $w = 2.50''$.

The formula for the area of a triangle is $A = \frac{1}{2}bh$ where b is the base and h the altitude upon that base. Using this formula, solve the following exercises:

5. Find A when $b = 12.4''$ and $h = 7.6''$.

6. Find h when $A = 48$ sq. in. and $b = 20''$.

7. Find b when $h = 6\frac{1}{2}''$ and $A = 32\frac{1}{2}$ sq. in.

8. Find A when $b = h = 2'\ 6''$.

DERIVATION OF FORMULAS

In each of the following cases derive a formula for the letter specified:

1. $A = lw$; $w =$

2. $V = Bh$; $B =$

3. $T = nc$; $c =$

4. $i = prt$; $t =$

5. $A = s^2$; $s =$

6. $V = \frac{1}{3}Bh$; $B =$

7. $C = \pi d$; $d =$

8. $A = p + i$; $i =$

9. $A = \pi r^2$; $r =$

10. $A = bh$; $b =$

11. $V = lwh$; $h =$

12. $i = prt$; $p =$

13. $s = \frac{1}{2}gt^2$; $g =$

14. $A = \frac{1}{2}bh$; $h =$

15. $A = \pi ab$; $a =$

16. $V = Bh$; $h =$

17. $A = lw$; $l =$

18. $V = \pi r^2 h$; $h =$

19. $VP = k$; $P =$

20. $V = \frac{1}{3}\pi r^2 h$; $h =$

21. $l = ar^{n-1}$; $a =$

22. $V = \frac{1}{4}\pi d^2 h$; $h =$

23. $T = nc$; $n =$

24. $i = prt$; $r =$

25. $V = lwh$; $w =$

26. $p = 2l + 2w$; $l =$

27. $A = p + i$; $p =$

28. $C = 2\pi r$; $r =$

29. $p = 2l + 2w$; $w =$

30. $i = A - p$; $p =$

31. $A = p(1 + rt)$; $p =$

32. $f + s + t = 180$; $f =$

33. $s = \frac{1}{2}gt^2$; $t^2 =$

34. $P = s - (c + e)$; $s =$

35. $V = \frac{1}{3}\pi r^2 h$; $r =$

36. $V = lwh$; $l =$

37. $l = a + (n-1)d$; $a =$

38. $s = \frac{1}{2}gt^2$; $t =$

39. $B = \frac{1}{14}lwt$; $w =$

40. $f + s + t = 180$; $s =$

41. $s = \frac{1}{2}n(a + l)$; $n =$

42. $T = 2\pi rh + 2\pi r^2$; $h =$

43. $l = a + (n-1)d$; $d =$

44. $e = f + v - 2$; $v =$

45. $P = s - (c + e)$; $e =$

After he has drawn the graph of a formula the student should see that he has done more than make a table of values and plot corresponding points. He should realize that what he has done is to draw an accurate picture of the relationship between the quantities which is expressed by the formula, and that from this picture he can frequently draw many interesting conclusions.

GRAPHS OF FORMULAS

1. On a section of squared paper like that below draw a cost graph for bananas at 25¢ per dozen.

Complete each of the following statements:

2. The graph in Ex. 1 shows that the cost of 5 doz. bananas is $ _____ .

3. The graph in Ex. 1 shows that _____ doz. bananas can be bought for $1.80.

4. The graph in Ex. 1 shows that doubling the number of dozens _____ the total cost.

5. The graph in Ex. 1 shows that the total cost of the bananas increases as the number purchased _____; that is, the cost to the purchaser depends upon the _____?
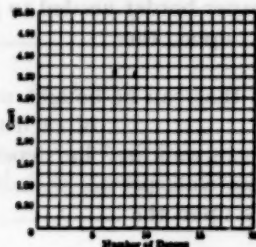
6. This graph of the formula $F = 1.8c + 32$, which may be used in changing temperature readings on the centigrade scale to readings on the Fahrenheit scale and vice versa, shows that a reading of 10°C. (centigrade) corresponds to a reading of _____ F.(Fahrenheit).

7. The graph in Ex. 6 shows that a reading of 32°F. corresponds to _____ C.

8. The graph in Ex. 6 shows that a reading of $-25°$C. corresponds to about _____ F.

9. A Fahrenheit reading is always 1.8 times the corresponding centigrade reading, plus _____.

10. Doubling a Fahrenheit reading does not double the corresponding _____ reading.

When one quantity depends upon another for its value, the mathematician says that the first is a *function* of the second. At this stage the idea of functionality must, of course, be approached informally, but it is important that the student should see that this valuable mathematical principle is of everyday occurrence in ordinary life.

DEPENDENCE OF QUANTITIES

In each of the blanks in the following statements insert the word which makes the best sense:

1. The cost of a sirloin steak depends upon the weight and the _____ per pound.
2. The value of the algebraic expression $5x - 3$ depends upon the value of _____.
3. The circumference of a circle depends upon the length of the _____ or of the _____.
4. The cost of sending a package by parcel post depends upon

the _____ of the _____ and the distance to
the place to which it is sent.

5. Doubling the length of the radius of a circle _____ the
circumference.

6. The number of yards of wall-paper border needed to go round
a rectangular room depends upon the _____ and
_____ of the room.

7. The number of theater tickets that can be bought with a 10-
dollar bill depends upon the _____ of each ticket.

8. The _____ that a boy can walk in 3 hr. depends upon
the number of miles that he can walk per _____.

9. The volume of a cube depends upon the _____ of an
_____.

10. The time that it takes me to fill in all the blanks on this page
at an average rate of 5 blanks per minute depends upon the
_____ of _____.

11. The interest received per year from an investment of $500
depends upon the _____ of interest at which the in-
vestment is made.

12. The cost of excavating a rectangular cellar at a fixed price per
cubic yard depends upon the _____, _____,
and _____ of the cellar.

13. Doubling the length of the radius of a circle multiplies the area
by _____.

The ability of the student to do the fundamental geometric con-
structions can be tested by a test like the one following.

FUNDAMENTAL CONSTRUCTIONS

*Using your ruler and compasses, draw a line which shall pass through
P and be perpendicular to l, in each of Exs. 1 to 4 inclusive. Leave all
construction lines on the page.*

1.   2.   3.   4.
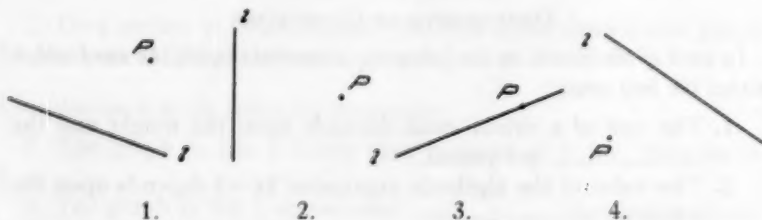
*Using only ruler and compasses, construct in each of these figures
the bisector of a given line segment or angle:*
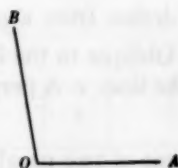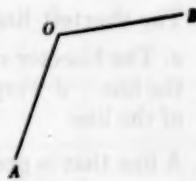
5.   **6.**   **7.**   **8.**

Using your ruler and compasses only, construct lines through point P which shall be parallel to line l. Leave all construction lines on the paper:
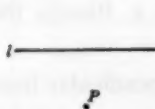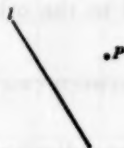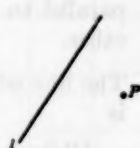
9.   **10.**   **11.**   **12.**

Using only your ruler and compasses, divide the lines in Exs. 13 to 16 inclusive into the number of parts specified. Leave all construction lines on the page:

13. *l* _____   **14.** *l* _____

      Three equal parts   Four equal parts

15. *l* _____   **16.** *l* _____

      Five equal parts   Six equal parts

Multiple-choice or best answer tests are good because they add variety and interest for the students who enjoy meeting the challenge by trying to select the best answer to each question.

INFERENCES REGARDING PERPENDICULARS

In each of the following exercises underline the expression which seem to you best for correctly completing each statement:

1. If two lines in a plane are each perpendicular to a third line, the two lines are

     a. Equal.   *b*. Complementary.   *c*. Perpendicular.   *d*. Horizontal.   *e*. Parallel.

2. If two adjacent sides of a parallelogram are perpendicular, the other two sides are

     a. Equal.   *b*. Supplementary.   *c*. Perpendicular.   *d*. Parallel.   *e*. Oblique.

3. The shortest line that can be drawn from a point to a line is

a. The bisector of the line.   *b.* Oblique to the line.   *c.* Parallel to the line.   *d.* Perpendicular to the line.   *e.* A perpendicular bisector of the line.

4. A line that is perpendicular to one of two parallel lines is

a. Parallel to the other.   *b.* Equal to the other.   *c.* Oblique to the other.   *d.* Equidistant from the other.   *e.* Perpendicular to the other.

5. A line that is perpendicular to one of two perpendicular lines

a. Is perpendicular to the other.   *b.* Intersects the other.   *c.* Is parallel to the other.   *d.* Is equal to the other.   *e.* Bisects the other.

6. The line which bisects the angle between two perpendicular lines is

a. Oblique to both lines.   *b.* Perpendicular to both lines.   *c.* Parallel to one of the lines.   *d.* Equal to both lines.   *e.* Parallel to both lines.

7. A line that is perpendicular to a slanting line

a. Bisects the slanting line.   *b.* Is parallel to the slanting line.   *c.* Is also a slanting line.   *d.* Is horizontal.   *e.* Is impossible.

8. A line that is parallel to one of two perpendicular lines

a. Is parallel to the other.   *b.* Is perpendicular to the other.   *c.* Is oblique to the other.   *d.* Bisects the other.   *e.* Is equal to the other.

9. In a right triangle two of the altitudes

a. Bisect each other.   *b.* Are perpendicular to the hypotenuse.   *c.* Are parallel.   *d.* Meet outside the triangle.   *e.* Coincide with the sides.

The test below is an ingenious type which is intended to test a complex of skills like that involved in solving a quadratic equation by the formula:

The Quadratic Formula

Taking $ax^2 + bx + c = 0$ as the type form of the general quadratic equation which has the two roots x_1 and x_2, insert the proper values in the following table, using the numbered columns which correspond with the eight given equations, as shown for the first five in Example 1:

1. $x^2 + 3x + 2 = 0$ 3. $x^2 + 9x - 36 = 0$ 5. $x^2 - 8x = -15$ 7. $x^2 - 10x = 11$
2. $x^2 - 3x + 2 = 0$ 4. $x^2 - 13x - 40 = 0$ 6. $x^2 + 10x = -24$ 8. $x^2 + 14x = 32$

	1.	2.	3.	4.	5.	6.	7.	8.
$a =$	1							
$b =$	3							
$c =$	2							
$-b =$	-3							
$b^2 =$	9							
$4ac =$								
$-4ac =$								
$2a =$								
$b^2 - 4ac =$								
$+\sqrt{b^2-4ac} =$								
$-\sqrt{b^2-4ac} =$								
$-b+\sqrt{b^2-4ac} =$								
$-b-\sqrt{b^2-4ac} =$								
$x_1 = \dfrac{-b+\sqrt{b^2-4ac}}{2a} =$								
$x_2 = \dfrac{-b-\sqrt{b^2-4ac}}{2a} =$								

Weaknesses of new-type tests. We should not conclude our discussion of the testing program without pointing out some of the possible weaknesses of new-type tests:

1. If care is not used, new-type tests may contain material of no possible importance.

2. Some of the tests are poorly arranged and badly printed. Some, made by busy classroom teachers, have had all sorts of impracticable features, such as loose detached sheets, transparent paper, and the like, that make them unfitted for classroom use.

3. Certain speed tests tend to glorify the machinery of mathe-

matics. Skill in mathematics should not be obtained at the expense of understanding. There is no value in merely finding out how fast errors can be made.

4. The tests do not measure attitudes, appreciation, and the like. This need not remain true, but it is so at the present time.

Conclusion. There is little doubt that the students themselves like the new types much better than the older ones. I know from long experience in making and giving tests that larger areas of subject matter may be tested in less time by the new-type tests and that the drudgery of scoring is greatly reduced by their use. I believe that we obtain more information about the extent and quality of a student's learning through the use of the newer tests and that remedial instruction is more intelligent and worth-while.

QUESTIONS AND TOPICS FOR DISCUSSION

1. Contrast prognostic and diagnostic tests with reference to their nature and purpose.

2. What do you consider the best kind of prognostic test for determining which students are likely to profit by a study of mathematics beyond the ninth grade?

3. State clearly what you consider to be the advantages and disadvantages of the "essay-type" examinations in mathematics.

4. What do you think are the advantages and disadvantages of "objective" type examinations?

5. Criticize the present marking system in your school. Can you suggest a scheme that is better and at the same time feasible?

6. Discuss the main arguments that seem to you valid against the use of "extramural" examinations.

7. Mention any strong or weak points of standardized tests that are not given in this article and that you feel to be worth considering.

8. Discuss the place of tests in curriculum construction, stating the purpose and nature of such tests.

9. Select some important objective which you wish to attain in teaching mathematics and make a test relating to it that shall be both valid and easily scored.

10. Discuss the importance of tests in providing a proper basis for remedial instruction.

11. What is the difference between what is commonly called a survey test and one that is diagnostic? Does either one include the other?

12. Try to obtain several standardized tests in mathematics and study the accompanying manuals in order to see what the purposes of each test are, and how the norms for it were established. Also discuss how each test is administered and scored.

WHEN WATER WORKS*

B. CLIFFORD HENDRICKS

457—24th Ave., Longview, Washington

"Water, water everywhere but not a drop to drink." So Alexander Selkirk complained as his shipwrecked boat drifted over the "briny sea." Trudy can agree with Selkirk about water everywhere but not as to its drinkableness. Her Clear Creek sky is cloud-covered more days than sunny. Those dripping, mist-breeding clouds furnish more than enough water. But to harness it for work; that is something else.

WEATHER WATER

Water furnished by weather has a way of its own. Varied are its amounts with time. Much, today and shortage tomorrow, maybe. Trudy knows that rain at Clear Creek is determined, in part, by wind direction. If winds come from the south and west, from the Pacific Ocean sixty miles away, fog, mist or rain may be expected. But if they come from the east, from over the mountains, the cooler tops of those mountains, generally, change the clouds to rain or snow before they reach Clear Creek. She also knows that wind changes are due to large areas of what are called "highs" or "lows" that drift over our North Temperate zone from west eastward. That the front of an eastward-moving "low" brings winds from the east and the back of it brings mist or rain-bearing winds from the west. "Highs," on the other hand, with their down-draft of cold heavier (denser) air usually clear the clouds away from their paths.

But "water for work" needs to come when needed. So at Trudy's home Mother Earth was called upon to meet the need. The fact that dependability, in this case, was found only after digging 300 feet below the surface seems to be one of Mother Nature's jokes. With so much water on top, why so far down to much again?

PUMPING THE PULL

Three hundred feet down gives the lifter quite a pull to get the water to the top. To pull that water to earth's surface, simple arithmetic tells us, it takes 300×0.44 pounds per square inch or a pull of 132 pounds. That is almost as much as the lift needed for three fifty-pound sacks of flour. And Trudy has also learned that if a pump is used (and that's the way water is usually moved) it will have to be a "force pump" since a "lift pump" can not pull the water higher than the push our air will give it. That push is only about 14.7 pounds per square inch so would hold up a column of water $(14.7 \div 0.44)$ or

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thirty-three feet high. That leaves 267 feet of water in her well to be pulled, or better pushed, to the top by what is called a "force pump." That pump, as are all pumps, is a machine and is used as a means of doing work upon the water. It puts the water where it can be made to do work for its users.

In addition to the pump another machine is used at Trudy's pump-house. This machine makes the pump's plunger-rod move up and down. The power for this rod-mover is an electric motor. Its trick of turning electric energy into pump-work will not be exposed just now but a helper in its job will be. The helper is really a "third class lever." The well-digger calls it a crank. By its movement about its center it lifts the rod up and down. This motion, by the aid of valves down 267 or more feet below, serves to get the water to the surface. The crank is rigidly fixed to a crank shaft that has a belt pulley at its other end. That pulley is made to turn round and round by a belt to it from the motor's pulley. In order to ease that pull, the motor has to exert, the pulley on it is smaller than that on the crank shaft. The radius of Clear Creek's motor pulley is 2.0 inches and that of the crank shaft pulley is 8.0 inches. Such an arrangement causes a pull of 33 pounds by the motor to provide a lift of 132 pounds on the plunger-rod. This four-fold multiplication of the motor's lift is called the "mechanical advantage" of the pair of pulleys. If one used a stop watch and counted the turns these two pulleys made in the same time he would find that the crank-shaft pulley goes only one fourth as fast as does the motor pulley. So mechanical advantage is here secured at the expense of speed. Note the mechanical advantage is computed by use of the two radii; eight inches is four times two inches. Note also when the advantage goes up the speed goes down; the ratios are direct and inverse.

PUTTING "PEP" INTO THE PIPES

If the water brought up were only for drinking purposes getting it to the surface would end the job. However, if the water is to do work for its users it must have more "pep" than gravity gives it at the surface level. In physics, water's "pep" is called pressure. Some people get this pressure by giving the water another lift above the ground level. They push it still higher up into a storage tank. As it is piped down from that "tank roost" gravity gives it pressure so that it can do work in various ways.

The water at Trudy's home is given "pep" in another way. In the pump house, aside from the motor and the crank drive for the plunger rod, there is a large closed tank. This tank is air tight and has air in it from the start. Since the air can not get out it is squeezed into smaller and smaller space as water is pumped under it. There is a

pressure gauge on the tank which tells how much the pressure of that "squeeze" is more than outside air-pressure. That means if one wishes to know how much the total "squeeze" on that trapped air is he has to add the gauge pressure to that read on the barometer for outside air pressure. At Trudy's home the pump keeps the tank's pressure at about twenty pounds per square inch. Thus if the barometer read thirty inches the "squeeze" pressure on the confined air would be 20.0+14.7 or 34.7 pounds per square inch. One cannot see the air in the tank so does not know, by observation, how much of the tank's room it takes but by the help of the gas law about volume change with pressure (Boyle's Law) he can readily compute what part of the tank's space is taken by the air.

There is another gadget attached to that tank. Before describing it let's recall that the pull the motor's plunger rod has to deliver, with the "squeeze" on the tank's air included, adds up to 132+34.7 or 166.7 pounds per square inch. If there were no let-up and the motor continued to push water into that tank the pressure-volume law tells us that motor's pull would get bigger and bigger. What would be the end of that? The electrician would say "That'll finally stall the motor." That is not wanted. So a small lever, probably first class, is attached with its effort arm hitched to a flexible membrane. That membrane is so connected to the tank's side that any change in the tank's air pressure makes it move out or in. The resistance arm of the lever is fastened to a switch that throws a circuit breaker when the pressure gets above twenty pounds or closes it when the pressure gets below twenty. Thus the motor runs when tank pressure is below twenty and restores the "pep," putting more water into the tank. This might be called "compression pep" in contrast with the other "gravity pep."

PIPING THE "PEP"

The water in this tank has "pep" because it has pressure. When it is allowed to flow out it loses that pressure. To keep its "pep" then its flow must be controlled; in a sense its "pep must be piped." Even in doing this simple machines of physics are used.

First some work jobs expected of "pepped" water at Trudy's house. There are more than the four to be listed. They are: watering and working the water heater; heating and flushing both the dish washer and the laundry tub; finally serving as scavenger for toilet cisterns and garbage dispenser.

All these machines are connected, through pipes, directly to the pressure and supply tank in the pump house. The water heater is heated electrically. The switch that controls the electric current to the heater is operated by a lever very similar to that described for

the pump house tank. In this case, however, the switch is thrown by a metal bar that bends due to unequal expansion of its two metal parts under the heat influence of the hot water in the tank. This sort of "thermo-regulator" will be more fully considered when the physics of heat is studied. The important fact in its operation is that metals, unlike gases as described by Charles Law, do not all expand to the same extent when heated. Just as the metal bar bends one way to shut off the current when the tank water gets too hot so it bends the other way to start the current (and heat) when it gets cooler. This performance keeps the hot water tank hot but at an even temperature so safe from explosive steam.

For both the dish washer and laundry tub the water flow or spray release is controlled by clockwork. The clock is a machine which by use of a pendulum-like fly wheel geared to the effort arm of a lever opens and closes a valve that permits water or spray to flow onto the articles to be washed. These valves are, in general, screw devices that by a turn or half turn shut the pipe opening into the tubs. The same "screw machine" is also used by any one who turns a pipe faucet to get water from it. Screws, it will be recalled, are simply long "inclined planes wrapped about a cylinder." The relation of the pitch (distance between threads) and the circumference of the screw's cylinder measures the screw's mechanical advantage.

SCAVENGER SERVICE

Some cisterns of the toilet room exhibit what might be called a compound lever; i.e. one lever operating upon another. One of these levers is "first class" and the other is "third class." The first lever serves to trip the other which opens the pipe for the flow of its water into the toilet stool. After the "flush" another lever automatically closes the refill pipe by an air-filled float serving to lift the effort end as the cistern's water level rises. Several simple problems on mechanical advantage and lever forces can be setup by making measurements of these lever arms.

After all the above services it would seem that the, now, almost "pepless," water should be permitted to peacefully join the weather-water of Clear Creek and ripple its way down to the mighty Columbia and out to the Pacific. Such is not to be for that part of the water doing scavenger service. Whether it is carrying garbage from the kitchen sink or the sewage from the bath room it is routed into a septic tank that time and quiet may unload its extras. Water makes two bids for this carrier job; it is a good dissolver of that to be removed or if dissolving won't do it, "water is good to float in." Archimedes' rule about floating helps to understand how water's, above average, density would keep most insoluble garbage fragments from sinking and so

failing to reach the septic basin. Then, too, this basin's slow-down of water's flow as it escapes keeps it from gouging into and carrying away rich top soil as it seeps more slowly into Clear Creek.

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS

REPORT OF THE NOVEMBER 25–27, 1954, CHICAGO CONVENTION

OFFICIAL ASSOCIATION MEETINGS

I. THE BOARD OF DIRECTORS MEETING

THURSDAY, NOVEMBER 25, 1954, 4:00 P.M. AND 7:30 P.M.— DINING ROOM 1, CONRAD HILTON HOTEL

1. *Roll Call:* Present: Officers: Price, Pella, Terhune, Soliday; Past-President Lauby; Directors: Edwards, Gibb, Junge, Marth, Otto, Panush, Shepard, Spangler; *Journal* Editor: Warner; Assistant Editor: G. Mallinson; Yearbook Editor: Shetler; Committee Chairman: Hunt, J. Mallinson.
2. *Minutes of Previous Meeting:* The reading of the minutes was dispensed with and the minutes as printed were approved.
3. *Special Reports:* (All reports were individually and formally received by the vote of the Board of Directors and filed with the secretary.)

    President Price reported on the association affairs, in particular stating that the report on revision of the by-laws would have to be published in two consecutive issues of the journal and could therefore not be brought before the membership for vote until the 1955 convention.

    The Local Arrangements Committee report was given by chairman Hunt.

    The *Journal* Editor's report was made by Dr. Warner; the *Yearbook* Editor's report by Shetler; the Historian's report by Terhune dealt with the Functions and Policies of the *Yearbook* Editor.

    Treasurer's report by Soliday resulted in the following action:

    a) It was moved by J. Mallinson, seconded by Gibb, that Mr. Soliday be authorized to dispose of the Anniversary books in any manner he sees fit, as long as this action does not result in a loss to the Association. Carried.
    b) It was moved by Junge, seconded by Otto, that the budget as submitted last May be approved. Carried.
    c) Mr. Soliday was authorized to invest up to $300 for more reprints to replenish exhausted supplies.
    d) Mr. Soliday was authorized to invest up to $200 to buy certain back numbers of the *Journal*, copies of which are no longer available for sale from the Association files.
    e) Authorization for renewal of insurance on back numbers of the *Journal* was voted. Motion included insuring the bound volumes of the *Journal* in Dr. Warner's keeping.
    f) The amount budgeted for the Cooperative Committee was raised from $35 to $50.
    g) It was moved by Junge, seconded by Panush, to delay action on the revision of the salary of the business manager until after the *Journal* Committee had time to consider it. Carried.

Edwards made the report as AAAS representative.

Jacqueline Mallinson reported on Emeritus Memberships.

Jones reported for the Place of Meeting Committee.

The meeting adjourned at 10:30 P.M.

II. ANNUAL BUSINESS MEETING

SATURDAY, NOVEMBER 27, 1954, 9:15 A.M—WALDORF BALLROOM

President H. Vernon Price, presiding, announced that up to this point there were 531 registrations for the convention. He expressed his appreciation for the work done by many people to make the convention a success, with particular recognition to Kenneth Hunt, chairman of Local Arrangements Committee; William Jones, chairman of Hotel Selection Committee; Past President Cecelia Lauby; Vice-President Milton Pella; and Treasurer-Business Manager, Ray Soliday.

Minutes of Previous Meeting:

It was moved, seconded, and carried that the reading of the minutes of the 1953 business meeting be dispensed with, since they were printed in the February, 1954, issue of the *Journal.*

Communications:

President Price read communications from E. S. Obourne of Paris, France, extending best wishes for a successful convention from the Science Teaching Section of Unesco, and from W. D. Reeve expressing regret at his inability to attend and extending his best wishes.

Place of Meeting:

Mr. William Jones, reporting for the Place of Meeting Committee, recommended Chicago as the convention city for 1956.

Emeritus Memberships:

Mrs. Jacqueline Buck Mallinson presented the name of Mr. Fred Nicolai for Emeritus membership.

President Price announced the resignation of Miss Ella Nichols from the Board of Directors.

Report of Nominating Committee:

Donald Lentz, Chairman, presented the following slate of candidates:

Milton O. Pella, President for 1954–1955
Charlotte Junge, Vice-President for 1954–55
Edward Bos, director through 1955, replacing Ella Nichols
E. Wayne Gross ⎫
Ralph C. Huffer ⎪ Directors through 1957
Clyde T. McCormick ⎬
Willard D. Unsicker ⎭

On request for nominations from the floor, Robert Lankton nominated Fred Leonhard for president. The nomination was seconded by Gerald Osborn.

Cecelia Lauby moved that the secretary be instructed to cast a unanimous ballot for the candidates for Vice-President and the Board of Directors. Seconded by Ray Soliday. Carried.

On motion, the nominations for president were closed and balloting for the two candidates was conducted by committee appointed by the president, consisting of Donald Lentz, Cecelia Lauby, and Robert Lankton.

As a result of the ballot, Milton O. Pella was elected President. It was moved by Louis Panush, seconded, and carried, that the secretary cast a unanimous ballot for Milton Pella for President. The newly elected officers were then introduced by President Price, and President-elect Pella responded.

Ray Soliday presented a financial report, stating that the past fiscal year had been the most financially profitable year in the history of the association.

The meeting adjourned.

III. BOARD OF DIRECTORS MEETING

SATURDAY, NOVEMBER 27, 1954, 1:00 P.M.—DINING ROOM 1

Roll Call:

President Price, Vice-President and President-Elect Pella, Past-President Lauby, Secretary Terhune, Treasurer-Business Manager Soliday, Directors (including electees) Bos, Edwards, Gibb, Gross, Junge, J. Mallinson, Marth, McCormick, Otto, Panush, Shepard, Spangler, Takala, and Unsicker.

President Price presided and introduced the new board members.

Journal Committee Report: Jacqueline Buck Mallinson

Reporting on the matter of the salary of the treasurer-business manager, since the association is on a sound basis for the first time in recent years, the committee recommended a $400 increase, instead of the $600 requested, with the provision that one year from now another review of the financial situation be made, and if possible at that time the additional $200 be granted. An additional recommendation was that the historian review the business manager's job. After much discussion it was moved by Junge, seconded by Otto, that the salary of the Treasurer-Business Manager include a $600 annual increment, retroactive to July 1, 1954, in addition to the base salary of $1200, subject to periodic review by the Board of Directors. Motion carried. Charlotte Junge moved, Edwards seconded, the acceptance of the report of the *Journal* Committee. Otto moved, Panush seconded, that the previous motion be amended to read "the report of the *Journal* Committee be received." Carried.

Policy and Resolutions Committee: No report.

Membership Committee: Robert Lankton

A copy of the report of the membership committee was placed on file. Marth moved, seconded by Spangler, that the report of the committee be received. Carried.

Historian's Report for 1955:

The historian was instructed on motion by Panush, seconded by Otto, to go back in the records to some strategic year and examine the historical development of the Treasurer-Business Manager's job. Carried.

With completion of the Old Business, President Price expressed his thanks for help given him during his administration, and turned the chair over to the new president Pella.

New Business:

The question of the status of the Geography Section was discussed. It was moved by Otto, seconded by Spangler, to discontinue the Geography Section due to apparent lack of interest. Carried. Moved by Otto, seconded by Spangler, that the Conservation Group be shifted from the Saturday morning session to the Friday afternoon session, that it be given section status, and that officers for this section for the 1955 convention be named by the president. Carried. Meeting adjourned at 3:15.

Respectfully submitted,
VIRGINIA TERHUNE, *Secretary*

Metallic yarns will be part of the automotive industry's upholstery this year, along with dresses of the same textile. Non-tarnishing and easily dry-cleaned, this metallic fabric is made by covering aluminum foil with a plastic laminate, that can be run on standard worsted or cotton looms as either warp or filling.

PROBLEM DEPARTMENT

Conducted by Margaret F. Willerding

Harris Teachers College, St. Louis, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution as on the following pages.

The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Willerding, Harris Teachers College, St. Louis, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solution should observe the following instructions.

1. Solutions should be in double spaced typed form.
2. Drawings in India ink should be on a separate page from the solution.
3. Give the solution to the problem which you propose if you have one and also the source and any known reference to it.
4. Each solution or problem for solution should be on a separate page.

In general, when several solutions are correct, the ones submitted in the best form will be used.

LATE SOLUTIONS

2417. *Walter Warne, St. Petersburg, Fla.*

2425, 2429. *J. Byers King, Denton, Md.*

2426. *L. V. Morris, Madison College, Tenn.*

2429. *L. R. Moenter, Fremont, Neb.*

2430. *C. W. Trigg, Los Angeles, Calif.*

2429. *Richard H. Bates, Milford, N. Y.*

Correction

2416. *Correction by C. W. Trigg, Los Angeles City College.*

In the solution published on page 759 of the December 1954 issue, as is evident from the figure, property 2) should read, "The orthogonal projection of the circumcenter on a side and the midpoint of the segment joining the orthocenter to the vertex opposite that side are the extremities of a diameter of the nine-point circle."

2431. *Proposed by C. W. Trigg, Los Angeles City College.*

The bearing of a ship which is x nautical miles from a second ship is $S\alpha°E$. If the second ship travels due west at U knots, in what time will the first ship traveling at V knots $(V>U)$ overtake her?

Solution by Leon Bankoff, Los Angeles, Calif.

Let y, z be the respective distances (in nautical miles) of the second and the first ships from their initial positions to the point of interception, and let t be the time required.

By the Cosine Law:

$$z^2 = y^2 + x^2 - 2xy \cos(90° + \alpha°)$$
$$= y^2 + x^2 + 2xy \sin \alpha°.$$

Substituting $y = Ut$ and $z = Vt$, we obtain

$$(Vt)^2 - (Ut)^2 - x^2 - 2xUt \sin \alpha° = 0$$

the positive root of which is

$$t = \left(\frac{x}{V^2 - U^2} \right) (U \sin \alpha° + \sqrt{V^2 - U^2 \cos^2 \alpha°}).$$

Solutions were also offered by V. C. Bailey, Evansville, Ind.; Alan Wayne, Brooklyn, N. Y.; Willis B. Porter, New Iberia, La.; Richard H. Bates, Milford, N. Y.; and the proposer.

2432. *Proposed by C. W. Trigg, Los Angeles City College.*

A cube is divided into two parts by the plane of a hexagon whose vertices lie on the edges of the cube, and divide these edges in the ratio $x : (1-1)$. Show (1) that the surface area of the cube is divided into the same ratio as the lengths of the edges and portions of edges in the two parts, (2) the perimeter of the hexagon is independent of x.

Solution by the proposer
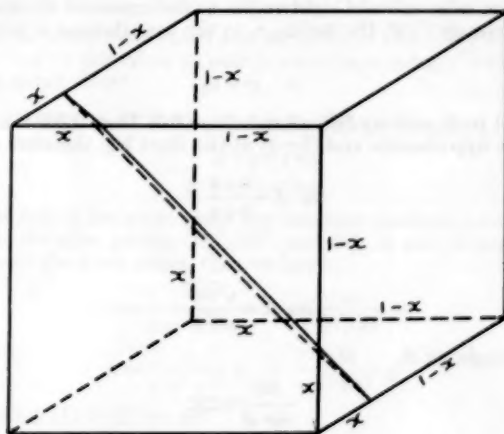
(1) Without loss of generality, we may work with a unit cube. It is evident from the figure that three faces are divided into parts with area $\frac{1}{2}(1-x)^2$ and $1 - \frac{1}{2}(1-x)^2$. The other three faces are divided into parts with areas $\frac{1}{2}x^2$ and $1 - \frac{1}{2}x^2$. Hence the ratio of the areas of the two parts of the cubical surface is

$$3[1/2(1-x)^2 + 1 - 1/2x^2] : 3[1 - 1/2(1-x)^2 + 1/2x^2]$$

or

$$(3 - 2x) : (1 + 2x).$$

The ratio of the lengths of the edges and portions of edges in the two parts is

$$3[1 + 2(1-x)] : 3[1 + 2x] \quad \text{or} \quad (3 - 2x) : (1 + 2x).$$

It will be observed that when the edges are bisected, $x = \frac{1}{2}$, the ratio becomes unity.

(2) The perimeter of the hexagon is $3\left[x\sqrt{2}+(1-x)\sqrt{2}\right]$ or $3\sqrt{2}$, which is independent of x.

It has been shown (Problem E897, *American Mathematical Monthly*, 57, 558, October 1950) that the area of the hexagon is $(1+2x-2x^2)\sqrt{3}/2$ and that the ratio of the volumes of the two parts is $(5-3x-3x^2+2x^3):(1+3x+3x^2-2x^3)$.

2433. *Proposed by Richard H. Bates, Milford, N. J.*

If an isosceles trapezoid is circumscripible, radius R, and also inscriptible, radius r,

(1) Show its area is

$$2r\sqrt{2r\sqrt{r^2+4R^2}-2r^2}.$$

(2) Show its diagonal is

$$\sqrt{2r\sqrt{r^2+4R^2}+2r^2}.$$

Solution by Armory R. Haynes, Tacoma, Wash.

Let the parallel sides of the trapezoid be $2a$ and $2b$, then the nonparallel sides are $(a+b)$. Radii to the vertices C and F bisect supplementary angles, therefore the non-parallel sides subtend right angles at the center of the inscribed circle. In the right triangle COF, the radius, r, to the hypothenuse is perpendicular to it as a tangent,

$$\therefore \quad r^2 = ab. \tag{1}$$

Produce HA to D, making $DH=b$ and draw BD. Then ABD is a right triangle with $(b+a)$ as hypothenuse and $(b-a)$ as the short leg, therefore C

$$\cos A = \frac{b-a}{b+a},$$

whence

$$\sin A = \frac{\sqrt{4ab}}{a+b}.$$

Now in triangle ACB,

$$\frac{BC}{\sin A} = 2R. \tag{2}$$

In the trapezoid the diagonals are equal and their product is equal to the sum of the products of the opposite sides:

$$\therefore \quad \overline{BC}^2 = 2a\times 2b + (a+b)(a+b)$$
$$BC = \sqrt{4ab+(a+b)^2}. \tag{3}$$

Substituting (3) in (2),

$$2R = \frac{(b+a)\sqrt{4ab+(a+b)^2}}{\sqrt{4ab}}$$

since $r^2=ab$, the diagonal,

$$BC = \sqrt{2r\sqrt{r^2+4R^2}+2r^2}.$$

The area is

$$S = (a+b) \cdot 2r = 2r(b+a) = 2r\sqrt{2r}\sqrt{r^2+4R^2} = 2r^3.$$ (1)

A solution was also offered by the proposer.
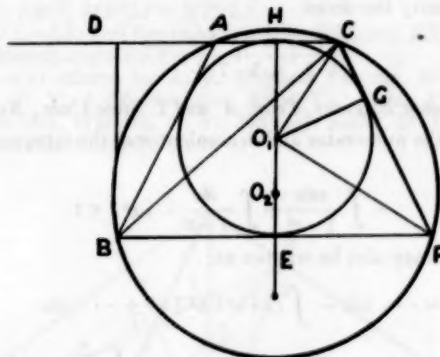
2434. *Proposed by Norman Anning, Alhambra, Cal.*

Show that the plane $x+y+z=0$ and the cylinder $x^2+xy+y^2=3$ have a circle in common. Find on this circle six points which have integral coordinates. What is interesting about them?

Solution by V. C. Bailey, Evansville College, Evansville, Ind.

$$x+y+z=0$$ (1)

$$x^2+xy+y^2=3.$$ (2)

The normal axis of the given plane has direction numbers $a=b=c=1$.

If we rotate the axes, getting OX', OY', and OZ', in such a way to make OZ' perpendicular to the given plane, then we have

$$x = \frac{-b}{\sqrt{a^2+b^2}} x' + \frac{-ac}{\sqrt{a^2+b^2}} y'$$

$$y = \frac{a}{\sqrt{a^2+b^2}} x' + \frac{-bc}{\sqrt{a^2+b^2}} y'$$ (3)*

By substituting (3) in (2) we get

$$x'^2 + y'^2 = 6.$$ (4)

Therefore, the plane and cylinder have a circle in common.

If (2) is solved for y by the quadratic formula, it is obvious from (1) and (2) that there are six and only six points on the circle of intersection with integral coordinates. They are

$$x = -1, \quad -1, \quad 1, \quad 1, \quad -2, \quad 2$$
$$y = -1, \quad 2, \quad 1, \quad -2, \quad 1, \quad -1$$
$$z = 2, \quad -1, \quad -2, \quad 1, \quad 1, \quad -1.$$

By using the distance formula one can show that these six points are the vertices of a regular hexagon.

A solution was offered by Sister Mary Paula, Baltimore, Md.

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* See Coordinate Solid Geometry by Bell.

2435. *Proposed by Lester Moskowitz, Brooklyn, N. Y.*

Use the integral

$$\int \frac{dx}{x^3+x+1}$$

to evaluate to infinity the series

$$\frac{1}{1\cdot2}+\frac{1}{4\cdot5}+\frac{1}{7\cdot8}+\cdots.$$

Solution by Clinton E. Jones, Tenn. A. and I. State Univ., Nashville, Tenn.

If we multiply the numerator and denominator of the integrand by $(x-1)$, we obtain,

$$-\int \frac{xdx}{1-x^3}+\int \frac{dx}{1-x^3}, \ |x^3| <1.$$

These integrals may also be written as:

$$\int (1+x^3+x^6+x^9+\cdots)dx - \int (x+x^4+x^7+x^{10}+\cdots)dx$$

$$= \int \sum_{n=1}^{\infty} x^{3n-3}dx - \int \sum_{n=1}^{\infty} x^{3n-2}dx.$$

If we now evaluate these integrals between zero and one, we have,

$$\sum_{n=1}^{\infty}\frac{1}{3n-2} - \sum_{n=1}^{\infty}\frac{1}{3n-1} = \left[1+\frac{1}{4}+\frac{1}{7}+\frac{1}{10}+\cdots\right]-\left[\frac{1}{2}+\frac{1}{5}+\frac{1}{8}+\frac{1}{11}+\cdots\right]$$

$$= \frac{1}{1\cdot2}+\frac{1}{4\cdot5}+\frac{1}{7\cdot8}+\frac{1}{10\cdot11}+\cdots$$

which is the series given above.

However, if we evaluate

$$\int_0^1 \frac{dx}{x^3+x+1}$$

in closed form we get $\pi/(3\sqrt{3})$. Therefore,

$$\frac{1}{1\cdot2}+\frac{1}{4\cdot5}+\frac{1}{7\cdot8}+\cdots = \sum_{n=1}^{\infty}\frac{1}{(3n-2)(3n-1)}=\frac{\pi}{3\sqrt{3}}.$$

A solution was also offered by the proposer.

2436. *Proposed by John Lowenthal, Exeter, N. H.*

In triangle ABC, AE is the altitude on BC, CD and BF meet AE at G. Prove that AE bisects angle DEF.

Solution by Leon Bankoff, Los Angeles, Calif.

DF cuts AE in L and BC in M. The line through L parallel to BC cuts FE in S and ED in T. Draw AM.

In the triangle ABC,

$$\frac{BE}{EC}\cdot\frac{CF}{FA}\cdot\frac{AD}{DB}=1, \text{ by Ceva's Theorem}$$

and

$$\frac{MB}{CM}\cdot\frac{CF}{FA}\cdot\frac{AD}{DB}=-1, \text{ by Menelaus' Theorem.}$$

Hence $BE/EC = -MB/CM$ and $MBEC$ is a harmonic range.

It follows that $MDLF$ is also a harmonic range; hence

$$MD/DL = MF/LF. \tag{1}$$

Now, in similar triangles TDL and DME, $MD/DL = ME/TL$, and in similar triangles FLS and FME, $MF/LF = ME/LS$.

Hence by (1), $TL = LS$; and the right triangles TEL and ESL are congruent. Therefore AE bisects angle DEF.

Solutions were also offered by A. R. Haynes, Tacoma, Wash.; Robert A. Atkins, Brooklyn, N. Y.; J. W. Neighbor, Aldie, Va.; Josiah Yerkes, Cortland, N. Y.; Peter V. Wilson, Watkins, N. Y.; Matie J. Smith, St. Petersbury, Fla.; and Willard Glazier, Hartford, Conn.

STUDENT HONOR ROLL

The Editor will be very happy to make special mention of classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each student contributor will receive a copy of the magazine in which his name appears.
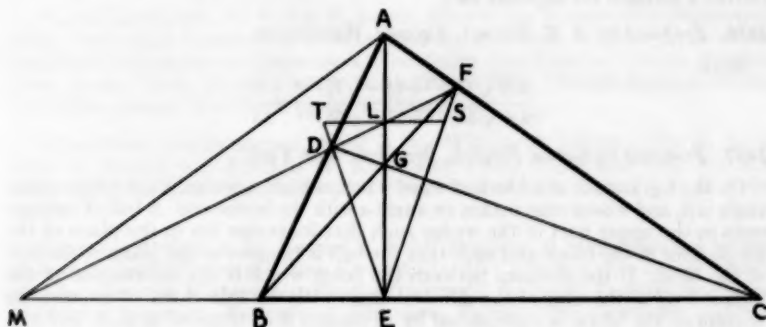
For this issue the Honor Roll appears below.

2436. *John Lowenthal, Exeter, N. H.*

PROBLEMS FOR SOLUTION

2449. *Proposed by Dewey C. Duncan, Los Angeles, Calif.*

For any positive rational value of $n > 1$, show

$$n^n > (n+1)^{n-1}$$

2450. *Proposed by C. W. Trigg, Los Angeles, Calif.*

Show how to dissect an equilateral triangle by straight cuts into four pieces which can be reassembled to form two triangles similar to the given triangle.

2451. *Proposed by Brother Felix John, Philadelphia, Pa.*

Express $46x^3 + 72x^2 + 18x - 11$ as the sum of two cubes.

2452. *Proposed by Brother Felix John, Philadelphia, Pa.*

In triangle ABC the perimeter is 36, t_b $4/3\sqrt{85}$ and $abc = 1530$. Find a, b, and c.

2453. *Proposed by C. W. Trigg, Los Angeles, Calif.*

(a) Show that every even integer greater than $12M+22$ can be expressed as the sum of M abundant numbers.

(b) Find the smallest integers which can be expressed as the sum of two abundant numbers in 1, 2, 3, 4, 5, 6, 7, 8, 9 ways.

2454. *Proposed by Leon Bankoff, Los Angeles, Calif.*

A quadrilateral with sides a, b, c, d is inscribed in a circle of radius R, in such a manner that the diagonals h and k are mutually perpendicular. Prove that $abcd = R^2(h^2+k^2-4R^2)$.

2455. *Proposed by Brother Felix John, Philadelphia, Pennsylvania.*

In triangle ABC, h_c, t_c, and m_c meet side c in points D, E and F respectively. Derive a formula for segment EF.

2456. *Proposed by A. R. Haynes, Tacoma, Washington.*

Show
$$\frac{(4+\sqrt{15})^{3/2}+(4-\sqrt{15})^{3/2}}{(6+\sqrt{35})^{3/2}+(6-\sqrt{35})^{3/2}} = \frac{7}{13}.$$

2457. *Proposed by Gerald Freilich, Brooklyn, New York.*

On the top surface of a block of wood h inches high, a wedge is cut whose plane angle is β, and whose edge makes an angle α with the horizontal. A ball of radius r rests in the upper part of the wedge such that its center lies in the plane of the top surface of the block and such that the ball is tangent to the plane of the side of the block. If the distance between the point which is the intersection of the higher level of the edge of the dihedral angle with the side of the block, and the bottom of the block is represented by d, express d in terms of α, β, h, and r.

2458. *Proposed by Julius S. Miller, New Orleans, La.*

A car, starting from rest, has a constant acceleration a and requires a velocity v, which it maintains for an interval; it then slows down to a stop at a constant rate b. If D is the total distance covered, show the total time to be
$$D/v+v/2(1/a+1/b).$$

2459. *Proposed by Paul D. Thomas, Norman, Okla.*

Show that
$$I = i \int^i e^{\pi i x} dx.$$

where $i=(-1)^{1/2}$, is a real number.

2460. *Proposed by Martin Hirsch, Brooklyn, New York.*

Two spheres with radii a and b, $a>b$, are glued together at a point P. The solid so formed is placed on a horizontal table and is made to roll, without slipping, on the table; prove that P describes a circle of radius r given by $r^2(a^2-b^2)=16a^3b^3$.

BOOKS AND PAMPHLETS RECEIVED

DEVELOPMENT OF THE GUIDED MISSILE, Second Edition, by Kenneth W. Gatland, F.R.A.S. Cloth. 292 pages. 14×21.5 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $4.75.

RELATIVITY FOR THE LAYMAN, by James A. Coleman, *Department of Physics and Astronomy, Connecticut College, New London, Connecticut.* Cloth. 131 pages. 13.5×20.5 cm. 1954. The William-Frederick Press, 313 West 35th Street, New York 1, N. Y. Price $2.75.

THE WHY OF CHEMISTRY PROBLEMS, by Fred B. Eiseman, Jr., *Chairman, Science Department, John Burroughs School, St. Louis, Missouri.* Cloth. 303 pages. 13.5×21.5 cm. 1954. Educational Publishers, Inc., 122 North 7th Street, St. Louis 1, Mo.

JET. THE STORY OF A PIONEER, by Sir Frank Whittle, K.B.E., C.B., F.R.S., Cloth. 320 pages. 13.5×21 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $6.00.

HUGH ROY CULLEN. A STORY OF AMERICAN OPPORTUNITY, by Ed Kilman, *Editor of the Houston Texas Post,* and Theon Wright, *Newspaperman and Writer.* Cloth. Pages viii+376. 14×22 cm. 1954. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. Price $4.00.

PLANE GEOMETRY, by Arthur F. Leary, *Head of the Mathematics Department, Hyde Park High School, Boston, Massachusetts,* and Carl N. Shuster, *Professor and Head of Mathematics Department, New Jersey State Teachers College, Trenton, New Jersey.* Cloth. Pages ix+510. 16.5×23 cm. 1955. Charles Scribner's Sons, 597 Fifth Avenue, New York 17, N. Y. Price $3.80.

FUNCTIONAL MATHEMATICS, Book 3, by William A. Gager, Lilla C. Lyle, Carl N. Shuster, and Franklin W. Kokomoor. Cloth. Pages xiii+481. 16.5×23 cm. 1955. Charles Scribner's Sons, 597 Fifth Avenue, New York 17, N. Y. Price $3.20.

ALGEBRA, COURSE 1, by Howard F. Fehr, *Professor of Mathematics, Teachers College, Columbia University, New York;* Walter H. Carnahan, *Assistant Professor of Education and Mathematics, Purdue University, Lafayette, Indiana;* and Max Beberman, *Associate Professor of Education, Florida State University, Gainesvilla, Florida.* Cloth. Pages xi+484. 15×23 cm. 1955. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $3.00.

ALGEBRA, COURSE 2, by Howard F. Fehr, *Professor of Mathematics, Teachers College, Columbia University, New York;* Walter H. Carnahan, *Assistant Professor of Education and Mathematics, Purdue University, Lafayette, Indiana;* and Max Beberman, *Associate Professor of Education, Florida State University, Gainesville, Florida.* Cloth. Pages x+502. 15×23 cm. 1955. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $3.00.

MATHEMATICS, A THIRD COURSE, by Myron F. Rosskopf, *Associate Professor of Mathematics Education, Teachers College, Columbia University;* Harold D. Aten, *Formerly Supervising Teacher of Mathematics and Counselor in the Public Schools of Oakland, California;* and William D. Reeve, *Professor Emeritus of Mathematics, Teachers College, Columbia University.* Cloth. 438 pages. 13.5×21.5 cm. 1955. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, N. Y. Price $3.48.

THE GYROSCOPE APPLIED, by K. I. T. Richardson, M.A. Cloth. 384 pages. 14.5×23 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $15.00.

THE DEVELOPMENT OF THE CONCEPT OF ELECTRIC CHARGE, by Duane Roller, *The American Association for the Advancement of Science,* and Duane H. D. Roller, *The University of Oklahoma.* Case 8. Paper. Pages iv+97. 15×23 cm. 1954. Harvard University Press, Cambridge, Mass. Price $1.60.

EDUCATION DIRECTORY, 1953–54. Part 4. Paper. 54 pages. 15×23 cm. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 30 cents.

SINGLE SIDEBAND FOR THE RADIO AMATEUR, prepared by the Headquarters Staff of the American Radio Relay League. Paper. 208 pages. 15.5×24 cm. 1954.

American Radio Relay League, West Hartford 7, Conn. Price $1.50 in the United States proper, $1.75 elsewhere.

BOOK REVIEWS

ESSAYS IN SCIENCE, by Albert Einstein. Cloth. Pages xi+114. 10×16.5 cm. 1934. Philosophical Library, 15 East 40th Street, New York 16, N. Y. Price $2.75.

This small pocket-sized book is, in a sense, a companion to another "The World As I See It." put out by the same publishers. Both are lifted from an original publication "Mein Weltbild" first published in Amsterdam.

This book has sixteen listings in its table of contents. The publishers call them essays though some are only one or two pages in length. Two are biographical in nature i.e. biographies of ideas rather than men; Johannes Kepler and Niels Bohr. A number are esssentially philosophical such as: Scientific Truth; Newton's Mechanics and later Theoretical Physics; Clerk Maxwell's Influence on the Idea of Physical Reality and the Problem of Space, Ether and the Field in Physics. Four chapters give verbal, rather than mathematical, attention to the theory of Relativity. Two chapters have to do with the application of physical principles to specific phenomena. They are headed: The Flittner Ship and River Meanders.

For the general reader, stymied by the mathematical barrier in an effort to know what Einstein has thought and done, this volume may offer help. However, ideas basic to his thinking are not necessarily made more lucid by mere translation of mathematical to verbal symbols. The critical reader of this offering is fully aware that lifting excerpts from an authors sayings entails competence of the one who makes the choices for their adequacy. Anthologies are attractive, in most cases, by reason of the reader confidence in their compilers. There is no citation of other than publisher responsibility for the choices made in this book.

B. CLIFFORD HENDRICKS,
Longview, Wash.

THE ZOO COMES TO YOU, by Captain Burr W. Leyson, *Assistant Curator, Department of Education, New York Zoological Society, New York,* and Ruth Manecke, *Educational Assistant at the Bronx Zoo, New York.* Cloth. 88 pages. 18×24 cm. 1954. E. P. Dutton and Company, Inc. 300 Fourth Avenue, New York 10, N. Y. Price $2.95.

New York Zoological Park, in 1944, started a unique service for school children. The slogan of this new venture in natural history extension might well be stated by "The Zoo Comes to You." As administered it provides a school lecture service with special focus upon: live exhibits carried by the speaker or film depiction of the animal subject in action.

The book, under review, is a sort of printed word and picture exhibit of some of the animals used in the spoken word and live exhibits of such school lectures.

The senior author has already created reader confidence by his former publications for non-technical readers. He only recently turned to the junior audiences by a first offering under the title "'Manty,' the Mantis!" He, in this book, is ably assisted by Miss Manecke who by her skill in photography, and perhaps in other ways, adds to its appeal.

By the help of twenty eight photograph reproductions, many full paged, stories of: The Little Owl; The Box Turtle; The White Rabbit; The Baby Deer; Petunia, the Skunk; Needles, the Porcupine and fifteen others, are told in an interest-holding fashion. There is no excessive bid for that interest by undue use of analogy or is the reader distracted by a strained effort "to keep to one syllable words."

The book is geared to a fourth or fifth grade reader though its pictures and orally told stories will hold even kindergarten or primary attention. The pictures

will, no doubt, prompt the youthful reader to return for many another look even though the story is not reread.

<div align="right">B. Clifford Hendricks</div>

College Algebra, by Paul K. Rees, *Associate Professor of Mathematics, Louisiana State University* and Fred W. Sparks, *Professor of Mathematics, Texas Technological College.* Third Edition. Cloth. 16.5×23.5 cm. Pages xiii+460. McGraw-Hill Book Company, Inc., 330 West 42nd St., New York 36, N. Y. 1954. Price $4.25.

In this revision the authors emphasize a rather extensive rewriting of material in attempt to achieve a more pleasing style and increased clarity of expression; the inclusion of some new material; a great variety of exercises; and a flexible order. In the reviewer's opinion, this book is definitely one of the better texts in college algebra which are available. One feature which seems is of value is the well graded lists of problems; the preface states that good coverage may be obtained by assigning every fourth problem—a spot check at various places seems to bear this out. An unusual feature is providing answers to three problems out of four, rather than half as is often the case. The explanations are very well written; the student is frequently warned against committing certain common mistakes. In general the standard of rigor in the definitions is good, for example in the treatment of fractional exponents certain cases are specifically excluded, with reasons given. (But sometimes there is a slip: on page 14 we find $a^0 = 1$ with no restriction, on page 117, with a cross reference to page 14, we find that a may not equal zero in this situation. There is no stated restriction on a in the law $a^m/a^n = a^{m-n}$).

At some places some instructors may disagree with the authors. These may be minor points, but they did not please this reviewer: apparently from page 285 the common logarithm of 1, 10, etc. does not have a mantissa; the abbreviation ml N for "the mantissa of the logarithm of N" is not in general use; from several points of view it is preferable to refer to the n-th term rather than the last term of a progression; on page 328 there seems to be no particular need for speaking of a series and then calling it a progression; it might be of value to present some problems concerning mathematical induction in which it is *not* possible to prove a formula, either because no true case can be found, or because it cannot be shown that the formula, true for k, is true for $k+1$; the American Experience Mortality Table has been largely replaced by more modern tables; the tables of trigonometric functions have nothing to indicate the omission of -10 from the logarithms, and with the use of the symbol for infinity (without any sign) one might conclude that log sin $0°$ is the same thing as log tan $90°$.

The points criticized are relatively minor; perhaps the feeling that the type was somewhat too small in the problem lists is an indication that age is creeping up on the reviewer. The entire impression made by the text was distinctly favorable—one would like to try this book as a text next year.

<div align="right">Cecil B. Read
<i>University of Wichita</i></div>

Introductory College Mathematics, by Chester George Jaeger, Ph.D., *Professor of Mathematics, Pomona College* and Harold Maile Bacon, Ph.D., *Professor of Mathematics, Stanford University.* Cloth. 16.5×24 cm. Pages xii+382. Harper and Brothers, 49 East 33d St., New York, N. Y. 1954. Price $4.75.

The authors state this book is planned not only for those who cannot afford more than one year of mathematics, but also for those who plan to go into work where more mathematics is required. Although the preface states that a class with strong high school preparation should be able to cover the material in a one-year course meeting thrice weekly, no doubt this will be insufficient time for many classes. The treatment is definitely not a casual glance at some topics in mathematics; in fact the discussion is in many spots more rigorous than the usual freshman text.

In a few spots the treatment is not entirely satisfactory: in the definition of

fractional exponents no restrictions whatever are placed on the quantities involved in the definition; the treatment of logarithms has some inconsistencies (p. 184 says "In *all* cases the mantissa is positive." This is hardly true for log 100; moreover on p. 185 we find ". . . this makes the . . . mantissa negative, whereas it is customary . . . to keep it positive.") A somewhat unusual definition on page 187 yields the information that the number 200 has only one digit. On page 337 it might be helpful to point out that a polar graph may be symmetrical to the polar axis without meeting the condition which is stated. On page 348 we read "Certain types of curves may be expressed more simply. . . ." Is it the curve, or the equation of the curve which is expressed more simply? Here also it might be valuable to point out that the parametric representation is not always equivalent to the direct functional relation.

By no means do the above criticisms imply that all is on the negative side of the ledger. There are many excellent features, to mention only a few: the excellent discussion of division by zero on pp. 16–17; the careful statement that a line parallel to the x axis has no slope; the fact that there may be a derivative with no value, or a value other than zero, and yet a maximum for a function; the pointing out of the fact that there is not universal agreement on the principal values of the inverse trigonometric functions.

Anyone interested in the book as a text obviously wants to know something of the content, without attempting to give a table of contents, some indication of the scope may be given by listing some topics covered: linear and quadratic functions; graphs, including statistical graphs; rates and limits; differentiation and integration of algebraic, trigonometric, exponential and logarithmic functions; areas and volumes; analytic trigonometry; plane analytic geometry, including the conic sections and such topics as tangents to conics; polar and parametric representation.

Cecil B. Read

SAMPLING TECHNIQUES, by William G. Cochran, *Professor of Biostatistics, School of Hygiene and Public Health, The Johns Hopkins University.* Cloth. 15.5 ×23.5 cm. Pages xiv+330. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. 1953. Price $6.50.

This book is planned not only as a text, but also as a reference for individual reading. It presupposes mathematical maturity equivalent to at least the calculus and a first course in mathematical statistics. The style is quite readable, the various chapters tend to form a separate unit, and illustrative material from actual surveys help the reader to grasp the material. There are many sets of exercises, with answers to a majority, which should aid the reader or form the basis of assignments in a formal course.

Possibly the reader whose major interest is in a field other than mathemaitcs will feel the book is too technical; however it is impossible to grasp the principles of the subject without some mathematical background, moreover it is possible to obtain much valuable information without covering all the proofs which are presented. As a single example one might consider the question of when one may use formulas valid for sampling from an infinite population in the case of a finite population, without introducing too much error.

For those interested in this aspect of statistics it would seem that this book is a very valuable addition to the existing literature.

Cecil B. Read

ANALYTIC GEOMETRY, SECOND EDITION, by Edward S. Smith, *Professor of Mathematics, University of Cincinnati;* Meyer Salkover, *Professor of Mathematics, University of Cincinnati;* and Howard K. Justice, *Assistant Dean, College of Engineering, Professor of Mathematics, University of Cincinnati.* Cloth. 15×21.5 cm. Pages xiii+306. John Wiley and Sons, Inc., 440 Fourth Ave., New York 16, N. Y. 1954. Price $4.00.

As is often the case with a new edition, teachers who found the first edition to

their liking will be glad to see a new edition; those who found the earlier edition unsuitable will no doubt feel the same about the second edition.

This should properly be classified as a rather complete text in analytic geometry, following largely a traditional treatment, with no use of calculus. The authors have used great care to be rigorous in their statements, and to avoid giving students a false impression. For example, they are careful in stating theorems to cover the case where the slope of a line does not exist; an example illustrates the fact that a pair of parametric equations may give only a portion of the graph of the corresponding cartesian equation.

Teachers interested in a very brief course could omit much material without loss of continuity, however, but for the somewhat doubtful advantage of additional reference material which would be available with this text, many will prefer a less comprehensive treatment. As illustrative of material frequently omitted, this book has material on conics from the point of view of sections of a cone; radical axis; maximum or minimum values for a quadratic function; invariants; tangents and normals; diameters of conics; pole and polar; higher plane curves; empirical equations; solid analytic geometry, including cylindrical and spherical coordinates.

The authors point out that this edition includes material on analytical proofs of geometric theorems and improved treatment of certain topics, such as asymptotes, angle bisectors, angle between two lines. A four place trigonometric table has been added. Certainly for the instructor seeking a book with rather full treatment of analytic geometry this is at least one text which must not be ignored when a change of text is anticipated.

CECIL B. READ

BIOLOGY FOR YOU, by B. B. Vance, *Chairman of Science and teacher of biology at the Daniel Kiser High School, Dayton, Ohio,* and *Assistant Professor of Biology and Education, University of Dayton, Dayton, Ohio;* and D. F. Miller, *Chairman of the Department of Zoology and Entomology, Ohio State University, Columbus;* and in consultation with W. R. Teeters, *Director of Education, St. Louis Public Schools, St. Louis, Missouri.* Cloth. 625 pages. Third Edition. 1954. 17×23.5 cm. J. B. Lippincott Company, New York.

This new edition of an already popular text has many additions and revisions which make it worthy of recommendation. The authors are trying to present biological information in an understandable language which should be a source of enjoyment to the average high school student. The book is designed for use in a course that is directly related to the past experiences and the future needs of the individual. Generalizations and broad understandings are stressed rather than treated as isolated facts.

The text is divided into fifteen units which include a preview, insects and man, common plants, living cells as the basis for life, the structure and function of plants, the major groups of plants, the animal kingdom, physical and mental health, the human body and its functions, disease, reproduction, heredity, evolution, conservation, and biology for vocations and hobbies. One of the outstanding features of the book is the wealth of illustrative material which is included. Six full color plates make the book especially attractive. The brightly colored molds and bacterial colonies found on page 23 were noted as being particularly good. Nearly every page has at least one clear-cut diagram or a good black-and-white photograph.

The authors made a special effort to use simple language, and when terms are used for the first time, they are explained or defined so that the pupil does not have to use the glossary excessively. Summaries and review questions are included at many points, and at the ends of each unit the authors have an extensive list of questions, some problems in scientific thinking, a list of suggested class and individual projects, and an outside reading list.

The generous glossary includes a pronunciation guide and contains many words which are not stressed in the text. This feature adds to the value of the

book as a reference work. It is probable that this volume will continue in wide usage for courses in biological science at the high school level.

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PAUL M. DANIELS
Oxford, Ohio

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SCIENCE FOR EVERYDAY USE, by Victor C. Smith, *Department of General Science, Ramsey Junior High School, Minneapolis;* and B. B. Vance, *Chairman, Science Department, Kiser High School* and *Associate Professor of Science and Education, The University of Dayton, Dayton;* in consultation with W. R. Teeters, *Director of Education, St. Louis Public Schools; St. Louis.* Third edition. Cloth. Pages xii +737. 22.5×15.5 cm. 1954. J. B. Lippincott Co. New York.

This new edition of a familiar science text includes up-to-date information on television, atomic energy, and jet aviation. The book is designed for use in either the one-year general science class taught at the eighth or the ninth grade level or for the third year of a three-year cycle of science which repeats the subject matter of the first two. The text contains an introduction and eighteen chapters which are grouped into six major units that include matter, energy, life, earth, man, and inventions. Each of the chapter headings is a statement which can easily be understood by the pupil and which summarizes the material to be found in the chapter. "The Earth's Surface Constantly Changes" is a chapter heading for the unit on Earth, for example. The introductory material for the chapters contains experiments, activities, and possible pupil reports which should serve as good motivators. The subject matter of the chapters is then divided into several titled sections which are of assignment length. Demonstrations with complete directions are included. Each section is followed by a study test which may be of several types but usually is of the fill-in type. Each chapter is followed by a word list, a list of related ideas, a review matching exercise, and thought questions. The text is illustrated throughout with diagrams and photographs, and several whole page full color illustrations add greatly to the attractiveness of the book. No workbook accompanies the text, and the authors recommend the use of a simple pupil notebook. A teacher's manual is available that lists sources of visual aid material, and there are reading lists at the end of each chapter and an extensive glossary at the end of the book.

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PAUL M. DANIELS

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NEW SCIENCE FELLOWSHIPS OFFERED AT
INDIANA UNIVERSITY

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Two special fellowship plans are now available to students pursuing the Master of Arts for Teachers degree in physical science. Applications for either fellowship are now available at the Office of the Director of Scholarships, Indiana University, Bloomington, Indiana. Completed applications should be sent to the Scholarships office not later than April 1, 1955. A special committee consisting of the Dean of the Graduate School and members of the faculty of the departments of Chemistry, Mathematics and Physics, and the School of Education will review applications and make the announcement of awards May 15, 1955.

THE STANDARD OIL FOUNDATION FELLOWSHIP PLAN. Five fellowships, each value at $200 and the remission of basic fees, will be available to candidates for the Master of Arts for Teachers degree (in Chemistry) who plan to enroll for courses for the Summer Session, 1955. A candidate for one of these fellowships must now be teaching or must be planning to teach chemistry in an Indiana high school.

DUPONT FELLOWSHIP PLAN. Two fellowships, each valued at $1200 and remission of fees, will be available to a candidate for the Master of Arts for Teachers degree (in Chemistry, Mathematics, or Physics) for the academic year 1955–56. The recipient of the Fellowship must be planning to teach science at the high school level.

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